Applied Physics - II
(MU)

Semester II – Common to All Branches

(Book Code – etME172)

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Chapter	Chapter Name
1	**Interference in Thin Film**
2	**Applications of Interference in Thin Film**
3	**Diffraction of Light**
4	**LASER**
5	**Fibre Optics**
6	**Electrodynamics**
7	**Charged Particle in Electric and Magnetic Fields**
8	**Nano-Science and Nanotechnology**

Interference in Thin Film

Syllabus Topic : Interference by Division of Amplitude and by Division of Wave Front

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Q.   What are the methods of coherent waves ?

Ans. :

Methods of coherent waves

(a) Division of wavefront

- When light from a source is allowed to pass through different two slits, the original wavefront gets partitioned into two wavefronts.

- The wavefronts travel through different paths and when they are united, they interfere.

(b) Division of amplitude

- The incident beam is divided into two or several beams by means of partial reflection at the surface of thin film.

- The amplitude and therefore the intensity of the original wavefront gets divided.

- This beams subsequently follow different optical paths

and produce interference when brought together.

- This method is known as division of amplitude.

Q.   Derive the conditions for maxima and minima due to interference of light reflected from thin film of uniform thickness. (Dec. 2013, Dec. 2014, May 2015, Dec. 2016)

Ans. :

- Consider a ray AB of monochromatic light of wavelength λ from an extended source incident at B, on the upper surface of a parallel sided thin film of thickness t and refractive index μ as shown in Fig. 1.3.1.

- Let the angle of incidence be i.

- At B, the beam is partly reflected along BR_1 and partly refracted at an angle r along BC.

- At C, it is again partly reflected along CD and partly refracted along CT_1. Similar partial reflections and refractions occur at points D, E etc.

- Thus we get a set of parallel reflected rays BR_1, DR_2 etc. and a set of parallels transmitted rays CT_1, ET_2, etc.

- For a thin film, the waves travelling along BR_1 and DR_2 in the reflected system will overlap.

- These waves originate from the same incident wave AB and hence are coherent.

- Hence they will interfere constructively or destructively according as the path difference between them is integral multiple of λ or odd multiple of $\frac{\lambda}{2}$.

(Reflected system)
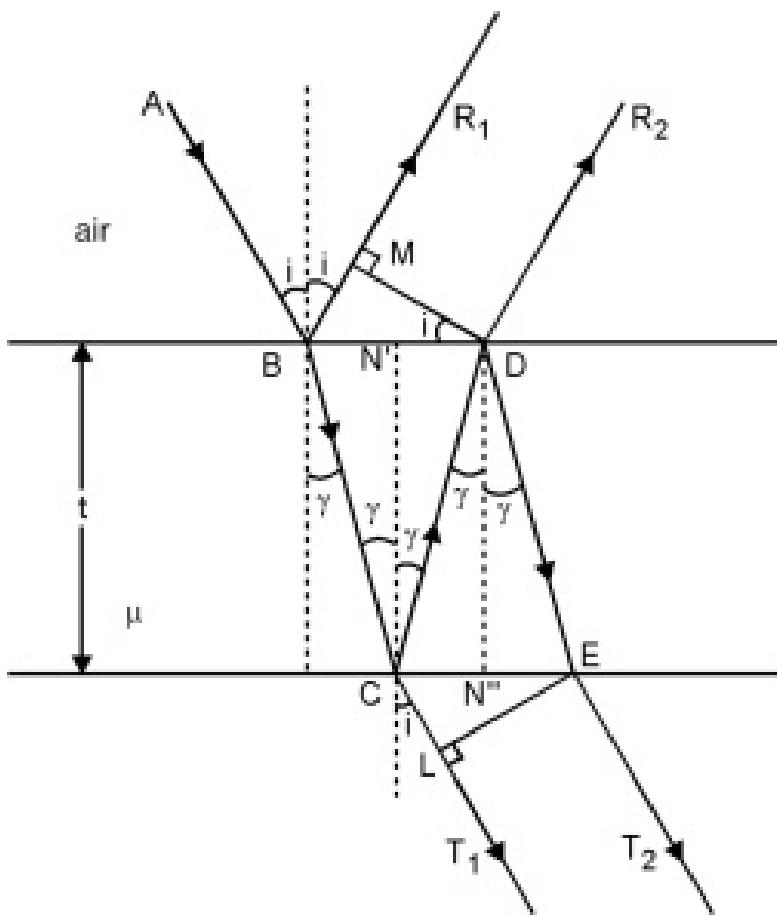
Fig. 1.3.1 : Interference in thin films

- To find the path difference between BR_1, and DR_2, draw DM perpendicular to BR_1. The paths travelled by the beams beyond DM are equal. Hence the optical path difference (optical path difference is obtained by multiplying geometrical path difference by its refractive index) between them is

    Δ = Path BCD in film − Path BM in air

    $= \mu \, (BC + CD) - BM$

- From Fig. 1.3.1 we have $BC = CD = \dfrac{t}{\cos r}$

    $\therefore \quad \mu \, (BC + CD) = \dfrac{2 \mu t}{\cos r}$

and $BM = BD \cdot \sin i = 2 BN \sin i$ ($\because BD = 2 BN$)
$= 2 t \tan r \sin i$ ($\because BN = CN \tan r = t \tan r$)
$\therefore \quad BM = 2t \dfrac{\sin r}{\cos r} \sin i$
$= \dfrac{2 \mu t}{\cos r} \sin^2 r$ ($\because \dfrac{\sin i}{\sin r} = \mu$)

∴ The optical path difference between the rays is

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r}\sin^2 r = \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

or $$\Delta = 2\,\mu\,t\,\cos r$$ ...(1.3.1)

- The film is optically denser than the surrounding medium air. Hence the ray BR_1 originating by reflection at the denser medium suffers a phase change of $\frac{\lambda}{2}$ due to reflection at B. (No such change of phase occurs for ray DR_2 as it is a result of reflection at C)

- Hence the effective path difference between BR_1 and DR_2 is

$$2\mu t \cos r + \frac{\lambda}{2}$$

Condition for maxima and minima in reflected light

(i) The two rays will interfere constructively if the path difference between them is an integral multiple of λ i.e.

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

or $2\mu t \cos r = (2n-1)\frac{\lambda}{2}$ where $n = 1, 2, 3, 4,$ (For maxima)

...(1.3.2)

or $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ when $n = 0, 1, 2, 3,$ ...

When this condition is satisfied the film will appear bright in the reflected system.

(ii) The two rays will interfere destructively if the path difference between them is an odd multiple of $\frac{\lambda}{2}$ i.e.

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or $$2\,\mu\,t\,\cos r = n\,\lambda$$ (For minima) ...(1.3.3)

where n = 0, 1, 2, 3, ...

Q. Why does an excessively thin film appear to be perfectly dark when illuminated by white light ? (May 2016)

Ans. :

The effective path difference between the interfering rays in reflected light is

$$2\mu t \cos r + \frac{\lambda}{2}$$

(i) If the film is excessively thin, then its thickness being very small as compared to the wavelength of light, the term $2 \mu t \cos r$ can be neglected as compared to $\frac{\lambda}{2}$. Hence the effective path difference becomes $\frac{\lambda}{2}$ which is the condition for minima. Hence every wavelength in the incident light will be absent in reflected system and the film will appear black in reflected light.

(ii) If the thickness of the film is large enough as compared to the wavelength of light, the path difference at any point of the film will be large. Under these conditions the same point will be a maximum for a large number of wavelengths and the same point will be a minimum for another set of large number of wavelengths. The number of wavelengths sending maximum intensity at a point are almost equal to the number of wavelengths sending minimum intensity. Also these wavelengths sending maximum and minimum intensity will be distributed equally over all colours in white light. Hence the resultant effect at any point will be the sum of all colours i.e. white in thick film.

(iii) Hence we define 'thin film' as : The film whose thickness is of the order of wavelength of the light which is used to expose it.

Syllabus Topic : Origin of Colours in Thin Films

Q. Why do we see beautiful colours in thin film when it is exposed to sunlight ?

(Dec. 2012, May 2015, Dec. 2016)

Ans. :

- When a thin film is exposed to white light from an extended source, it shows beautiful colours in the reflected system.

- Light is reflected from the top and bottom surfaces of a thin film and the reflected rays interfere.

- The path difference between the interfering rays depends on the thickness of the film and the angle of refraction r and hence on the inclination of the incident ray.

- White light consists of a continuous range of wavelengths. At a particular point of the film and for a particular position of the eye (i.e. t and r constant) those wavelengths of incident light that satisfy the condition for the constructive interference in the reflected system will be seen in reflected light.

- The colouration will vary with the thickness of the film and inclination of the rays (i.e. with the position of the eye with respect to the film). Hence if the same point of the film is observed with an eye in different positions or different points of the film are observed with the eye in the same position, a different set of colours is observed each time.

Syllabus Topic : Wedge-Shaped Film (angle of wedge and thickness measurement)

Q.   Why are the fringes in wedge shaped film straight ? Derive the conditions of maxima and minima for interference in wedge shaped films ?   (Dec. 2013, May 2014, Dec. 2015)

Ans. :

Fringes in wedge shaped film straight because

- Consider a film of non-uniform thickness as shown in Fig. 1.7.1. It is bound by two surfaces OX and OX′ inclined at an angle θ. The thickness of the film gradually increases from O to X.

- Such a film of non-uniform thickness is known as wedge shaped film. The point O at which the thickness is zero is known as the edge of the wedge.

Conditions of maxima and minima for interference in wedge shaped films

- The angle θ between the surfaces OX and OX' is known as the angle of wedge. Let be the refractive index of the material of the film.

- Let a beam AB of monochromatic light of wavelength λ be incident at an angle i on the upper surface of the film. It is reflected along and is transmitted along BC. At C also the beam suffers partial reflection and refraction and finally we have the ray DR_2 in the reflected system.

- Thus as a result of partial reflection and refraction at the upper and lower surfaces of the film, we have two coherent rays BR_1 and DR_2 in the reflected system.
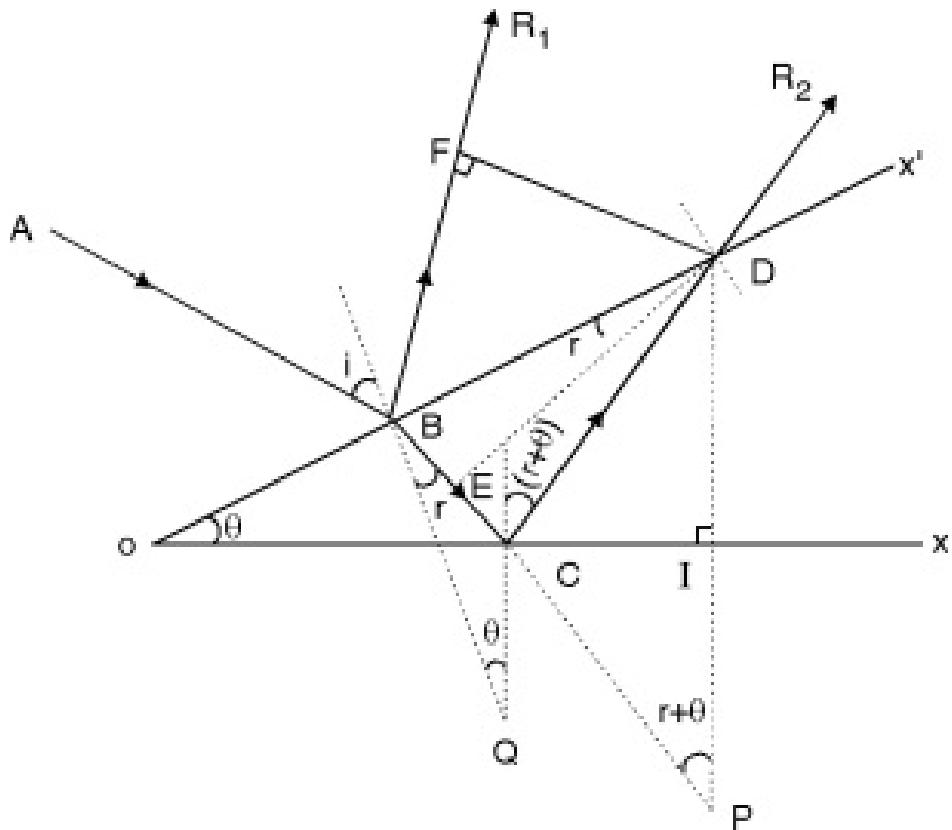
Fig. 1.7.1 : Interference in wedge shaped film

- These rays are not parallel but diverge from each other. These rays interfere constructively or destructively according to whether the path difference between them satisfies the condition for constructive or destructive interference in the reflected system.

- To find the path difference between these two rays, draw DF perpendicular to BR_1.

- The optical path difference between the rays BR_1 and DR_2 is

  $$\Delta = \mu\,(BC + CD) - BF \qquad \qquad \text{...(1.7.1)}$$

- Draw a perpendicular to surface OX at point C.

- We have the perpendicular to OX′ at point B. Both these perpendiculars will meet at point Q as shown in diagram.

  $$\therefore \angle BQC = \angle XOX' = \theta$$

  Draw a perpendicular DE on BC from D.

  As θ is small enough, BE = EC

  Also from diagram $\angle QBE = r = \angle BDE$

- Draw a perpendicular from D on OX such that intersects OX at I and BC produced at P. Also we get CP = CD.

- Equation 1.7.1 can be written as

  $$\Delta = \mu\,(BC + CD) - BF = \mu\,(BE + EC + CP) - BF$$

  From diagram

  $$\mu \;=\; \frac{\sin i}{\sin r} \;=\; \frac{BF}{BE} \qquad \text{or} \qquad BF \;=\; \mu\,BE$$

  $$\therefore \quad \Delta = \mu\,(BE + EC + CP) - \mu\,BE = \mu\,EP \quad (\text{as } E - C - P)$$

- Now consider ΔDPC

  CP = CD, ∠ CPD = r + θ

  and Δ DPE is a right angle triangle

  $$\therefore \cos (r + \theta) = \frac{EP}{DP}$$

  $$\therefore \quad EP = DP \cos (r + \theta) = 2\, t \cos (r + \theta)$$

  where DP = 2 DI = 2t , t = thickness of film at point D

  $$\therefore \quad \Delta = \mu\, EP = 2\, \mu\, t \cos (r + \theta) \qquad \qquad ...(1.7.2)$$

- Due to reflection at B, an additional path change of $\frac{\lambda}{2}$ occurs for the ray BR_1. Hence the total path difference between the interfering rays is $2 \mu t \cos (r + \theta) + \frac{\lambda}{2}$

- Hence for maxima, we have the condition for constructive interference

  $$2 \mu t \cos (r + \theta) + \frac{\lambda}{2} = n\lambda$$

  $$\text{or} \quad 2 \mu t \cos (r + \theta) = (2n - 1)\frac{\lambda}{2} \qquad \qquad ...(1.7.3)$$

  n = 1, 2, 3, 4 .....

- For minima, we have the condition for destructive interference

  $$2 \mu t \cos (r + \theta) + \frac{\lambda}{2} = (2n - 1)\frac{\lambda}{2}$$

  $$2\, \mu t \cos (r + \theta) = n\lambda \qquad \qquad ...(1.7.4)$$

  n = 0, 1, 2, 3, ...

- It is clear that for a maximum or a minimum of a particular order, t must remain constant. In case of the wedge shaped film, t remains constant along lines parallel to the thin edge of the wedge.

- Hence the maxima and minima are straight lines parallel to the thin edge of the wedge.

- At the thin edge, t = 0 hence path difference between the rays is $\frac{\lambda}{2}$, a condition for darkness. Hence the edge of the film appears dark. It is called as **zero order band**. That is the reason extensively thin film appears dark (black) in reflected light.

- Beyond the edge for a thickness t for which path difference is λ, we obtain the first bright band. As t increases to a value for which path difference is $\frac{3\lambda}{2}$ we obtain the first dark band.

- Thus as the thickness increases we obtain alternate bright and dark bands; which are equally spaced and equal in width.

(i) For normal incidence and air film, r = 0 and μ = 1

$$\text{Total path difference} = 2t \cdot \cos\theta + \frac{\lambda}{2}$$

$$2t\cos\theta = (2n - 1)\frac{\lambda}{2}, \quad \text{For maxima}$$

and $2t\cos\theta = n\lambda$ For minima

(ii) For very small angle of the wedge,

As $\theta \to 0$, $\cos\theta \to 1$

∴ For constructive interference,

$$2t = (2n - 1)\frac{\lambda}{2}$$

and for destructive interference

$2t = n\lambda$

∴ For constructive interference

$$t = (2n - 1)\frac{\lambda}{4} \quad n = 1, 2, 3, \ldots$$

$$t = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

i.e. **every next bright fringe will occur for thickness interval of** $\frac{\lambda}{2}$ **each.**

Similarly for destructive interference

$$t = \frac{n\lambda}{2} \quad n = 0, 1, 2, 3, \ldots$$

$t = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$

i.e. every next dark fringe will occur for thickness interval of $\frac{\lambda}{2}$ each.

Syllabus Topic : Newton's rings

Q. Explain why Newton's rings are unequally spaced ? (May 2012)

OR

Q. Show that the diameter of Newton's n^{th} dark ring is proportional to square root of ring number. (May 2015)

Ans. :

Newton's rings

- When a plano-convex lens of large radius of curvature is placed on a plane glass plate, an air film is formed between the lower surface of the lens and upper surface of the plate.

- The thickness of the film gradually increases from the point of contact outwards.

- If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings, with their centre dark, is formed in the air film.

- These rings were first studied by Newton and hence known as Newton's rings. They can be seen through a low power microscope focussed on the film.

Diameter of dark and bright rings

- Let POQ be a plano-convex lens placed on a plane glass plate AB. Let R be the radius of curvature of the lens

surface in contact with the plate. Fig. 1.9.2.
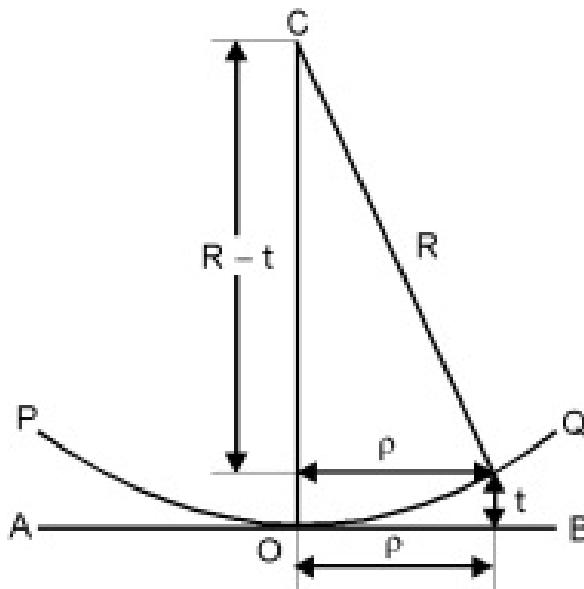
Fig. 1.9.2

- Let ρ be the radius of a Newton's ring corresponding to the constant film thickness 't'. The path difference between the two interfering rays in the reflected system is $2\mu t \cos (r + \theta) + \dfrac{\lambda}{2}$.

λ = wavelength of incident light.

μ = 1   for air film.

r = 0   for normal incidence

θ = 0   for large R.

$\therefore$ Path difference = $2t + \dfrac{\lambda}{2}$ ...(1.9.1)

From Fig. 1.9.2 we see that

$R^2 = \rho^2 + (R - t)^2$

or $\rho^2 = R^2 - (R - t)^2$

or $\rho^2 = 2Rt - t$

$t \ll R$ and hence we have

$\rho^2 = 2Rt$

$$\therefore \quad 2t = \frac{\rho^2}{R} \qquad \qquad ...(1.9.2)$$

$\therefore$ Path difference between the interfering rays is $\frac{\rho^2}{R} + \frac{\lambda}{2}$.

For dark rings

- The condition to get dark rings is that

$$\text{Path difference} = \frac{\rho^2}{R} + \frac{\lambda}{2}$$

$$= (2n + 1)\frac{\lambda}{2} \quad (n = 1,2,3,....)$$

- If D is the diameter of Newton's ring, then $\rho = \frac{D}{2}$

$$\therefore \quad \frac{D_n^2}{4R} = n\lambda \text{ where } D_n = \text{Diameter of } n^{th}\text{ dark ring.}$$

$$\therefore \quad D_n^2 = 4nR\lambda$$

$$\mathbf{D_n = \sqrt{4nR\lambda}} \qquad \qquad ...(1.9.3)$$

$$D_n \propto \sqrt{n}$$

- Hence the diameter (and hence radius) of the dark ring is proportional to the square roots of natural numbers. One can say that rings are unequally spaced, because difference between two consequtive rings represents thickness of ring.

For bright rings

- The condition to get bright rings is that the path difference

$$\frac{\rho^2}{R} + \frac{\lambda}{2} = n\lambda$$

or $$\frac{\rho^2}{R} = (2n - 1)\frac{\lambda}{2} \text{ where } n = 1, 2, 3....$$

$$\therefore \quad \rho^2 = (2n - 1)\frac{\lambda R}{2}$$

Putting $\rho = \frac{D}{2}$, we have,

$$\frac{D_n^2}{4} = (2n - 1)\frac{\lambda R}{2}$$

Where, D_n = Diameter of nth bright ring

$$D_n^2 = 2\lambda R \cdot (2n - 1)$$
$$D_n = \sqrt{(2n - 1)}\sqrt{2\lambda R} \qquad \qquad ...(1.9.4)$$
$$D_n \propto \sqrt{2n - 1} \qquad \text{where } n = 1, 2, 3, ...$$

- Hence the diameter (and hence radius) of the bright ring is also proportional to the square root of odd natural numbers.

Q. Why centre of Newton's rings is always dark ? (May 2014)

Ans. :

- At the point of contact the thickness of air film is zero. Consider the case of thin film at this point i.e. condition for bright spot is given by,

  $2 \mu t \cos (r + \theta) = (2 n + 1) \lambda/2$ $n = 0, 1, 2, ...$

  and condition for dark spot is given by,

  $2 \mu t \cos (r + \theta) = n\lambda$

- For Newton's ring set up r = 0 (large radius of lens $\theta = 0$), (for normal incidence r = 0)

  $\therefore \quad \cos (r + \theta) = \cos 0 = 1$

  $\therefore \quad$ Condition for bright

  $2 \mu t = (2n + 1) \lambda/2$ $n = 0, 1, 2, 3, ...$

- Condition for dark

  $2 \mu t = n\lambda$

- At point of contact, t = 0 and one can see that condition for bright does not get satisfied. But condition for dark gets satisfied for t = 0 and n = 0.

Q. Why Newton's rings are circular and wedge shaped films are straight ? (May 2014, Dec. 2016)

Ans. :

- In both air-wedge film and Newton's ring experiments, each

fringe is the locus of points of equal thickness of the film. In Newton's rings arrangement, the locus of points of equal thickness of air film lie on a circle with the point of contact of plano-convex lens and the glass plate as centre. So, the fringes are circular in nature and concentric.

- For wedge shape air film, the locus of points of equal thickness are straight lines parallel to the edge of the wedge. So, fringes appears straight and parallel.

Q.   Suppose that in experiment on Newton's Rings, first light of red colour is used and then blue light, which set of rings would have larger diameter ? Justify your answer with proper expression. (May 2013)

Ans. :

Newton's Ring with White Light

- If a monochromatic light source is used in Newton's ring experiment alternate dark and bright rings are obtained. If we use white light instead of monochromatic light, a few mixed coloured rings around a black centre are observed and beyond it, a uniform illumination is obtained.

- White light consists of seven colours of different wavelength. Diameters of the Newton's ring is proportional to the wavelength of the different colours. As we know that $\lambda_r > \lambda_v$, therefore the diameter of violet ring of the same order will be smallest and those for red ring will be the largest, and the diameters of other coloured rings shall occupy the intermediate positions.

- Due to overlapping of the rings of different colours over each other, only first few coloured rings will be clearly seen

while other rings cannot be observed.

Solved Problems

Ex. 1.8.1 (Dec. 2015)

Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 Å. Calculate the angle of wedge in seconds of an arc.

Soln. :

Given : $\mu = 1.52$, $\lambda = 5893 \times 10^{-8}$ cm, $\beta = 1$ mm $= 0.1$ cm.

Formula : The fringe spacing is

$$\beta = \frac{\lambda}{2\mu\theta}$$

The angle of wedge in radians is

$$\theta = \frac{\lambda}{2\mu \cdot \beta} = \frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.1} \text{ radians}$$

$$\therefore \quad \theta = \frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.1} \times \frac{180}{\pi} \times 3600 \text{ seconds}$$

$$\therefore \quad \theta = 40 \text{ seconds of an arc.} \quad \text{....Ans.}$$

Ex. 1.15.3 (Dec. 2014)

Light of wavelength 5880 Å is incident on a thin film of glass of $\mu = 1.5$ such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it dark by reflection.

Soln. :

$\lambda = 5880 \times 10^{-8}$ cm, $\mu = 1.5$, $r = 60°$, $t = ?$

Condition for film to appear dark is,

$2 \mu t \cos r = n\lambda$

The smallest thickness will be for n = 1.

$$2 \times 1.5 \times t \times \cos 60 = 1 \times 5880 \times 10^{-8}$$

$$t = \frac{5880 \times 10^{-8}}{2 \times 1.5 \times 0.5}$$

$$= 3920 \times 10^{-8} \text{ cm} \qquad \text{...Ans.}$$

Ex. 1.15.8 (Dec. 2012)

Light falls normally on a soap film of thickness 5×10^{-5} cm and of refractive index 1.33. Which wavelength in the visible region will be reflected most strongly ?

Soln. :

The condition of maxima is given by,

$$2 \mu t \cos r = (2n - 1) \lambda/2 \qquad \text{where n = 1, 2, 3, ...}$$

Here,   Given :   t = 5×10^{-5} cm μ = 1.33

$$r = 0° \qquad \text{i.e. } \cos r = 1$$

Now, $\lambda = \dfrac{4 \mu t \cos r}{(2n - 1)} = \dfrac{4 \times 1.33 \times 5 \times 10^{-5}}{(2n - 1)}$

By substituting the values of n = 1, 2, ... we get a series of wavelengths which shall be predominantly reflected by the film.

For n = 1, $\lambda_1 = \dfrac{4 \times 1.33 \times 5 \times 10^{-5}}{1} = 26.66 \times 10^{-5} \text{ cm}$

Similarly,

For n = 2, $\lambda_2 = 8.866 \times 10^{-5}$ cm

For n = 3, $\lambda_3 = 5.32 \times 10^{-5}$ cm

For n = 4, $\lambda_4 = 3.8 \times 10^{-5}$ cm

Out of these wavelengths 5.320×10^{-5} cm lies in the visible region (4000 Å to 7500 Å).

Hence 5320 Å is the most strongly reflected wavelength. ...Ans.

Ex. 1.15.13 (May 2013)

A plane wave of monochromatic light falls normally on a uniform thin film of oil, which covers a glass plate. The wavelength of the source can be varied continuously. Complete destructive interference is obtained only for wavelengths $5000 \, \overset{\circ}{A}$ and $7000 \, \overset{\circ}{A}$. Find the thickness of the oil layer. Given R.I. of oil = 1.3 and R.I of glass = 1.5.

Soln. :

Here the path is from air to oil and oil to glass.

For air to oil

For destructive interference the condition is, (for normal incidence)

$$2\mu_{oil} \, t_{oil} = (2n + 1)\frac{\lambda}{2} \qquad \ldots(1)$$

As the arrangement remains same for both the wavelengths μ_{oil} and t_{oil} will be the same or constant.

$$\therefore \quad const. = (2n + 1)\frac{\lambda}{2}$$

$\therefore$ Order of destructive interference is inversely proportional to wave length.

$\therefore$ For λ_1 = 7000 Å take order n and for next wavelength λ_2 = 5000 Å take next order i.e. n + 1

For λ_1 = 7000 Å

$$2\mu_{oil} t_{oil} = (2n + 1) \times \frac{7000}{2} \, \text{Å} \qquad \ldots(2)$$

For λ_2 = 5000 Å

$$2\mu_{oil} t_{oil} = (2(n+1) + 1) \times \frac{5000}{2} \, \text{Å} \qquad \ldots(3)$$

$$(2n + 1) \times \frac{7000}{2} = (2(n+1)+1) \times \frac{5000}{2}$$

$$\frac{2n+1}{2n+3} = \frac{5000}{7000}$$

On solving, n = 2

$\therefore$ Substitute in Equation (2)

$$t_{oil} = \frac{(2(2)+1) \times 7000}{2 \times 1.3 \times 2} = 6730.769 \, \text{Å} \qquad \ldots \text{Ans.}$$

Ex. 1.15.18 (May 2016)

The diameter of 5th dark ring in Newton's ring experiment was

found to be 0.42 cm. Determine the diameter of 10th dark ring.

Ans. :

$D_n^2 = 4n\lambda R$

As Diameter of 5th dark ring $= 0.42$ cm

Now diameter of 10th dark ring $= ?$

$\dfrac{D_{10}^2}{D_5^2} = \dfrac{4 \times 10 \times \lambda R}{4 \times 5 \times \lambda R} = \dfrac{2}{1}$

$D_{10} = \sqrt{2}\,(D_5) = \sqrt{2}\,(0.42)$

$= 0.594$ cm

$\therefore$ Diameter of 10th dark ring $= 0.594$ cm

$\rightarrow\rightarrow\rightarrow$

Ex. 2.2.1

Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges in sodium light at normal incidence, what is the thickness of the wire?   (Given λ = 5893 Å)

Soln. :

Fringe width $\beta = \dfrac{\lambda}{2\mu\theta}$ ...(For normal incidence)

For air film $\mu = 1, \ \beta = \dfrac{\lambda}{2\theta}$

Thickness of wire $t = x_n \tan \theta$

$= x_n \cdot \theta$ (for small θ, $\tan \theta \simeq \theta$)

$= 20 \cdot \beta \cdot \theta$
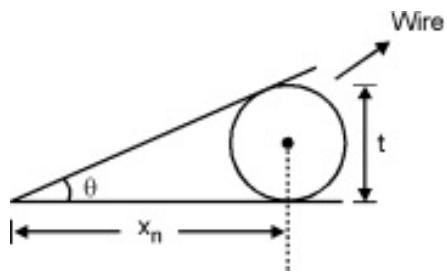
Fig. P. 2.2.1

As there are 20 fringes between two edge points

Putting the value of β, we get,

$$t = 20\left(\frac{\lambda}{2\theta}\right) \cdot \theta = 10\,\lambda$$

$= 10 \times 5893 \times 10^{-10} = 5.893 \times 10^{-6}$ m

$= 5.893\ \mu m$ 　　　　　　　　　　　...Ans.

Syllabus Topic : Wavelength of Incident Light

Q.   With proper diagram and necessary expressions explain how Newton's ring experiment is useful to determine the radius of curvature of plane convex lens.

(May 2013, Dec. 2014)

Ans. :

- A carefully cleaned convex surface of a plano-convex lens L of large radius of curvature is placed on a plane glass plate P. (Fig. 2.3.1).
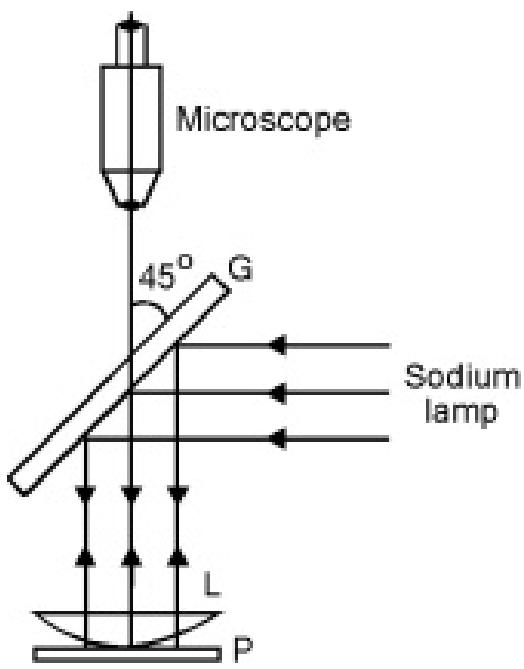
Fig. 2.3.1

- Another glass plate G is held at a suitable distance above at an angle 45° with the vertical (to make the normal incidence on the film).

- With a condensing lens, maximum light from source (Sodium lamp) is allowed to fall on G. The inclined plate G reflects light onto the air film between the lens L and plate P.

- Newton's rings are formed as a result of interference between the rays reflected from the top and bottom faces of the air film.

- They are seen through a low power microscope focussed on the air film where the rings are formed.

- The effective path difference between the interfering rays is

$$2\mu t \cos(r + \theta) + \frac{\lambda}{2}$$

where μ = R.I. of film.

θ = Angle of film at any point

λ = Wavelength of light

- If D_n is the diameter of nth dark ring then as per the theory of Newton's rings described in Equation (1.9.3),

$$D_n^2 = 4nR\lambda$$

where R = Radius of curvature of lower surface of lens.

Let $D_{n+p}^2 = 4(n+p)R\lambda$ for in a pth ring ...(2.3.1)

$\therefore D_{n+p}^2 - D_n^2 = 4pR\lambda$

$\lambda = \dfrac{D_{n+p}^2 - D_n^2}{4pR}$ ...(2.3.2)

- Alternately if wavelength of incident monochromatic ray is known, using Equation (2.3.2), we can find R i.e. the radius of curvature.

- For more accurate approach we plot a graph of D_n^2 vs n as shown in Fig. 2.3.2.

- Thus by measuring the diameter of nth and $(n + p)^{th}$ dark rings and the radius of curvature R, the wavelength λ can be calculated.

- The diameter of the rings is measured with the travelling microscope and the radius of curvature can be determined by using lens equation or by a spherometer.

Fig. 2.3.2

Determination of Radius of Curvature of Lens
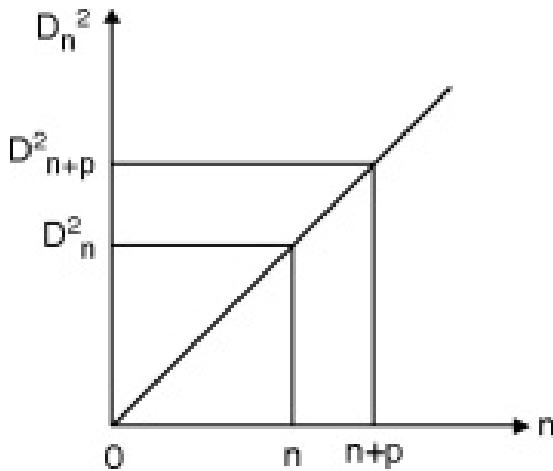
- This can be done easily with the help of spherometer and the formula:

$$R = \frac{l^2}{6h} + \frac{h}{2} \qquad \qquad ...(2.3.3)$$

Where l = distance between two legs of spherometer.

h = Difference in the reading when placed on lens as well as when placed on surface.

Ex. 2.3.1 (May 2015)

In Newton's rings experiment the diameter of 5th ring was 0.336 cm and the diameter of 15th ring was 0.590 cm. Find the radius of curvature of plane-convex lens if the wavelength of light used is 5890 Å.

Soln. :

Given : $D_5 = 0.336$ cm, $D_{15} = 0.590$ cm, $\lambda = 5890$ Å $= 5890 \times 10^{-8}$ cm

Formula : $D_{n+p}^2 - D_n^2 = 4pR\lambda$.

$$R = \frac{D_{n+p}^2 - D_n^2}{4 \cdot p \cdot \lambda} = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}} = 99.91 \text{ cm}$$

∴ $R = 99.91$ cm.

Q. How is Newton's ring experiment used to determine refractive index of liquid medium ? (May 2014, May 2016)

Ans. :

- There is a popular branch of engineering known as "

Hydraulics" in which liquid is in motion and transmits energy, Viscosity and refractive index are some of the parameters which can be used to decide the suitability of the oil (Called Grade of the oil). In such applications, the correct measurement of the R.I. is thus an important issue.

- Consider that a transparent liquid whose refractive index is to be determined is placed between the lens L and plate P of the Newton's rings arrangement.

- If the liquid is rarer than glass, a phase change of π will occur at the reflection from the lower surface of liquid film.

- If the liquid is denser than glass, then a phase change of π will occur due to reflection at the upper surface of the film.

- Hence in either case, a path difference of $\lambda/2$ will be introduced between the interfering rays in the reflected system; and hence the effective path difference between them will be $2\mu t \cos(r + \theta) + \dfrac{\lambda}{2}$

Now $\qquad\qquad\qquad\qquad$ r = 0 for normal incidence

$\qquad$ θ = 0 for large R

$$\therefore \text{ Path difference } = 2\mu t + \frac{\lambda}{2}$$

$$2t = \frac{\rho^2}{2}$$

∴ For nth dark ring, we have,

$$\frac{\mu \rho^2}{R} + \frac{\lambda}{2} = (2n + 1)\lambda/2 \qquad \therefore \frac{\mu \cdot D_n^2}{4R} = n\lambda$$

$$\therefore \qquad D_n^2 = \frac{4nR\lambda}{\mu}$$

Similarly for the (n + p)th dark ring we have,

$$D_{n+p}^2 = \frac{4(n+p)R\lambda}{\mu} \qquad\qquad \dots(2.4.1)$$

$$D_{n+p}^2 - D_n^2 = \frac{4pR\cdot\lambda}{\mu}$$

$\qquad$ D$_{n + p}$ = Diameter of (n + p)th dark ring, then we have

$$\mu = \frac{4pR\lambda}{\left(D_{n+p}^2 - D_n^2\right)_{liquid}} \qquad\qquad \dots(2.4.2)$$

$$\therefore \qquad \mu = \frac{4pR\lambda}{\left[D_{n+p}^2 - D_n^2\right]_{air}} = 1 \qquad (\because \mu = 1 \text{ for air}) \qquad \dots(2.4.3)$$

For more accurate approach we plot a graph of D_n^2 vs n as shown below

$\therefore$ From Equations (2.4.2) and (2.4.3) we have,

$$\mu = \frac{\left[D_{n+p}^2 - D_n^2\right]_{air}}{\left[D_{n+p}^2 - D_n^2\right]_{liquid}}$$

- If we obtain ring structure using air as the medium then diameter is given by D_n^2 (air) $= 4nR\lambda$. Now slowly if we insert a liquid with R.I. μ then

- D_n^2 (liquid) $= \dfrac{4nR\lambda}{\mu}$, Take ratio of these two.

$$\mu = \frac{D_n^2 \text{ (air)}}{D_n^2 \text{ (liq)}} \qquad \qquad \ldots(2.4.4)$$

- Thus to find μ, Newton's rings are formed with air film. The diameters of nth and $(n + p)^{th}$ dark rings are measured for air film.

- Then the transparent liquid is introduced between lens L and plate P.

- The diameters of nth and $(n + p)^{th}$ dark rings are measured with liquid film. Hence using Equation (2.4.4), μ can be calculated.

Syllabus Topic : Testing of Surface Flatness

Q.   Write a short note on : Determination of Optical Flatness.

Ans. :

Determination of Optical Flatness
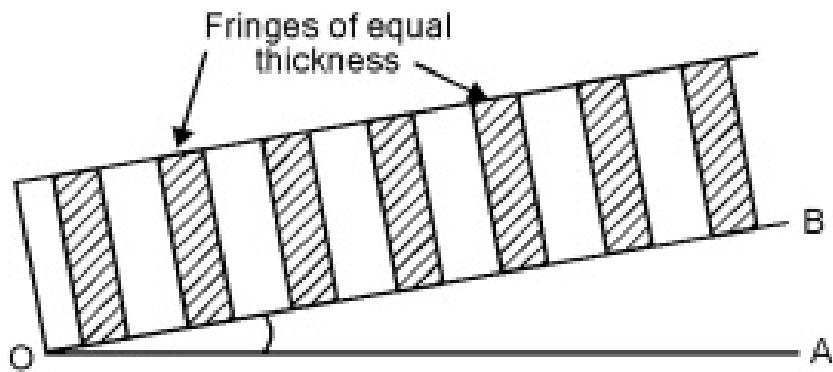
Fig. 2.5.1

- The phenomenon of interference is also used in testing the plainness of the surfaces.

- If two surfaces OA and OB (Fig. 2.5.1) are perfectly plane, the air film between them gradually varies in thickness from O to A. The fringes are of equal thickness as each fringe is the locus of the points at which the thickness of the film has a constant value.

- If the fringes are not of equal thickness, it means that the surfaces are not plane.

- To test the optical flatness of a surface, the specimen surface to be tested (OB) is placed over an optically plane surface (OA).

- The fringes are observed in the field of view. If they are of equal thickness the surface OB is plane. If not, then surface OB is not plane.

- The surface OB is polished and the process is repeated. When fringes observed are of equal width, it means the surface OB is plane.

- The accuracy in this method is far more superior compare to any other technique adopted. The accuracy level is of the order of fraction of micrometer.

Syllabus Topic : Anti Reflecting Films

Q. Describe in detail the concept of anti reflecting film with a proper ray diagram of thin film interference. Which condition the material should satisfy to act as anti reflecting film? (May 2012)

Q. What do you understand by anti-reflecting coating ? Derive

Ans. :

Anti-reflecting coating

- We are aware that compound microscope, telescope, camera lenses etc. uses a combination of lenses.

- When the light enters the optical instrument at the glass air interface, around 4% of light (for air with $n_1 = 1$ and glass with $n_2 = 1.5$) that too at single reflection is lost by reflection which is highly undesirable. For advance telescopes the total loss comes out to be nearly 30% and can not be tolerated if working under low intensity applications.

- In order to reduce the reflection loss, a transparent film of proper thickness is deposited on the surface. This film is known as "non Reflecting film".

- Popular material used is MgF_2 because its refractive index is 1.38 (i.e. between air and glass). Cryolite ($n_1 = 1.36$) is also used.

- Thickness of the film may be obtained for given purpose as shown below :
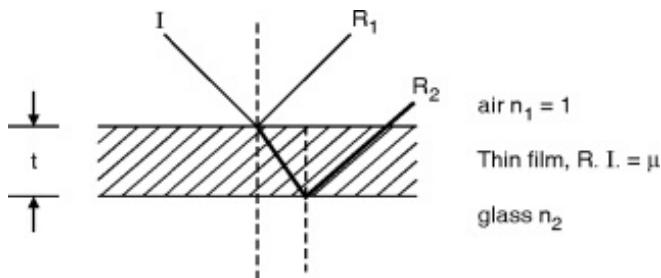
Fig. 2.6.1 : Thin film coating

- Let a ray I incident up on thin film of MgF_2 coated on glass.

This ray is reflected from upper surface as R_1 and from lower surface as R_2. The optical path difference between these two rays is n_1 (2t). As the incident ray enters from rarer to denser twice i.e. at air to film and film to glass.
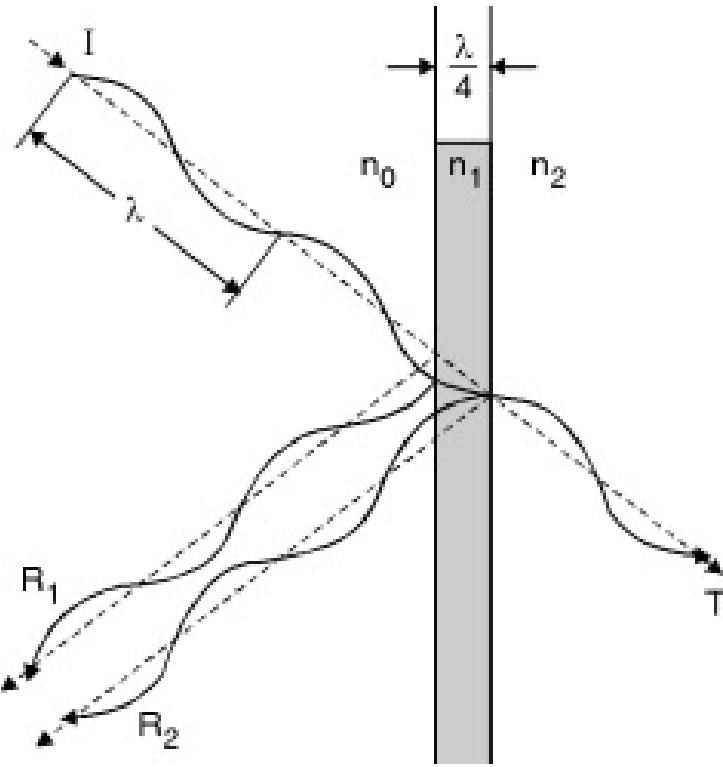
Fig. 2.6.2

- If both the rays R_1 and R_2 interfere with each other and path difference is $(2n + 1) \lambda/2$ (for n= 0, 1, 2, ...) then destructive interference will take place.

$$\therefore \quad 2 n_1 t = \frac{\lambda}{2} \qquad \text{(for n = 0)}$$

$$\therefore \quad n_1 t = \frac{\lambda}{4 \mu}$$

- It means, in order to have destructive interference a layer of $n_1 t = \frac{\lambda}{4}$ is coated on glass plate.

Syllabus Topic : Highly Reflecting Film

Q. Write short note on : Highly Reflecting Film.

Ans. :

Highly Reflecting Film

- As shown in the case of non Reflecting film, we have seen that a thin film of thickness $\lambda/4$ will create additional path difference of $\lambda/4 + \lambda/4 = \lambda/2$ or additional phase difference $\frac{\pi}{2} + \frac{\pi}{2} = \pi$.

- This creates destructive interference.

- The same logic is extended by considering a film of thickness $\lambda/2$.
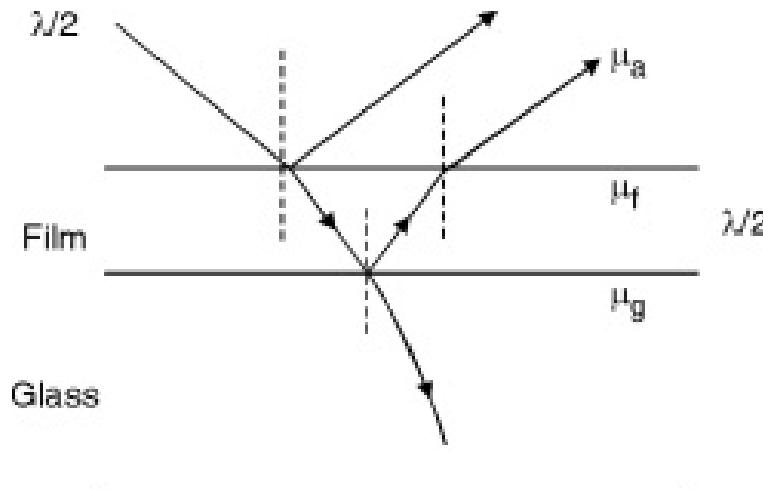
Fig. 2.7.1

- In this case the total path difference is $\frac{\lambda}{2} + \frac{\lambda}{2} = \lambda$ or phase difference of $\pi + \pi = 2\pi$.

- Thus by the condition of complete constructive interference is satisfied. With this we can make majority of light reflected back from the surface of the glass.

- This principle is used in all kind of sun control films used on car, sun glasses etc.

Solved Problems

Ex. 2.8.1 (May 2013, May 2014, 3 Marks)

In a Newton's rings experiment, the diameter of the 5th ring was 0.336 cm and that of 15th ring was 0.59 cm. If the radius of curvature of the plano - convex lens 100 cm. Calculate the

wavelength of light.

Soln. :

Given : R = 100 cm, D_{15} = 0.59 cm, D_5 = 0.336 cm

Formula : $\lambda = \dfrac{D_{15}^2 - D_5^2}{4 \times n \times R} = \dfrac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 100}$

$= \dfrac{0.2352}{4000} = 5.88 \times 10^{-5}$ cm   ...Ans.

Ex. 2.8.3 (Dec. 2012, 5 Marks)

In a Newton's ring experiment the diameter of the 10th dark ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

Soln. :

D_n (air) = 1.4 cm

D_n (liquid) = 1.27 cm

$\therefore$ $\mu = \dfrac{D_n^2 \text{ (air)}}{D_n^2 \text{ (liquid)}} = \dfrac{1.4^2}{1.27^2} = 1.215$ ...Ans.

Ex. 2.8.4 (Dec. 2012, 5 Marks)

Newton's rings are formed using light of wavelength 5896 Å in reflected light with a liquid placed between plane and curved surface. The diameter of 7th bright fringe is 0.4 cm and radius of curvature is 1 m. Find the refractive index of liquid.

Soln. :

For Newton's rings diameter of nth dark ring with air as medium

$$D_n^2 = 4nR\lambda$$

And with liquid having R.I. μ is given by

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$\therefore \quad \mu = \frac{4nR\lambda}{D_n^2} = \frac{4 \times 7 \times 1 \times 5896 \times 10^{-10}}{(0.4 \times 10^{-2})^2}$$

$$= 1.038 \qquad \qquad \text{...Ans.}$$

Ex. 2.8.9 (Dec. 2016, 5 Marks)

In Newton's ring experiment the diameter of nth and $(n + 8)^{th}$ bright rings are and 7 mm respectively. Radius of curvature of lower surface of lens is 2 m. determine the wavelength of light use.

Soln. :

Given :

(1)   Diameter of nth bright ring = 4.2 mm

(2)   Diameter of $(n + 8)^{th}$ bright ring = 7 mm

(3)   Radius of curvature of plane convex lens = 2 m

To find : Wavelength of monochromatic light.

Formula :

$$D_n^2 = 2\lambda R (2n - 1)$$

For $(n + 8)^{th}$ ring

$$D_{n+8}^2 = 2\lambda R (2(n + 8) - 1) = (7 \times 10^{-3})^2 \qquad \text{...(1)}$$

For n^{th} ring

$$D_n^2 = 2\lambda R (2n - 1) = (4.2 \times 10^{-3})^2 \qquad \text{...(2)}$$

∴   Divide Equations (1) by (2)

$$\frac{2n + 15}{2n - 1} = \left(\frac{7}{4.2}\right)^2 = 2.7778$$

∴   n = 5

Using this in Equation (2)

$$D_n^2 = 2\lambda R(2n-1) = (4.2 \times 10^{-3})^2$$

$$\therefore \quad \lambda = 4.9 \times 10^{-7} \text{ m} \qquad \text{...Ans.}$$

Ex. 2.8.11 (May 2014, 4 Marks)

Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.01 mm is introduced between them at one end to form an air film. If the light used has wavelength 5900Å, find the separation between consecutive bright fringes.

Soln. :
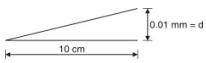
Fig. P. 2.8.11

Let l = 10 cm,

d = 0. 01 mm

λ = 5900 Å

here, $\tan \theta = \dfrac{0.01 \times 10^{-3}}{10 \times 10^{-2}} = 1 \times 10^{-4}$

For very small angles, $\tan \theta \approx \theta$

Now fringe width $\beta = \dfrac{\lambda}{2\mu\theta}$

For air film, $\mu = 1$

$$4\beta = \dfrac{5900 \times 10^{-8}}{2 \times 1 \times 10^{-4}} = 0.295 \text{ cm}$$

$\therefore$ Separation between two consecutive bright fringes is 0.295 cm   ...Ans.

➔➔➔

Diffraction of Light

Q.   What is meant by diffraction? State its types and differentiate them. (Dec. 2015)

Ans. :

Diffraction

The phenomenon of bending of light round the corners of an obstacle and spreading of light waves into the region of geometrical shadow of the obstacle is called diffraction.

Types of Diffraction

The phenomenon of diffraction is divided into two types :

1.   Fresnel diffraction

2.   Fraunhoffer diffraction

Difference between Fresnel diffraction and Fraunhoffer diffraction

Sr. No.	Fresnel diffraction	Fraunhoffer diffraction
1.	Distance of source and screen from obstacle is finite.	Source and screen are at infinite distance from obstacle.
2.	No lenses are required to study diffraction in the laboratory.	Two biconvex lenses are required to study diffraction in laboratory.
3.	The wavefront incident on the aperture is either spherical or cylindrical.	The wavefront incident on aperture is plane.
4.	The diffracted wavefront is either spherical or cylindrical.	The diffracted wavefront is plane.
5.	The initial phase of secondary wavelets is different at different points in the plane of aperture.	The initial phase of secondary wavelets is same at all points in the plane of aperture.
6.	It has no importance in optical instruments.	It is very important in optical instruments.

Q.   Explain Fraunhoffer Diffraction at a Single Slit.

Ans. :

Fraunhoffer Diffraction at a Single Slit

Step I

- Consider a single slit of width 'a' illuminated by monochromatic light of wavelength λ as shown in Fig. 3.4.1.

- The incident plane wavefront is diffracted by the slit and is then focussed on the screen by the lens L.

- Every point of the incident wavefront in the plane of the slit acts as a secondary source and sends out secondary waves in all directions.

- The secondary wavelets travelling normally to the slit are brought to focus at point P_O by the lens.

- All these secondary waves travel the same distance through the lens along the direction $\theta = 0$ and hence produce maximum intensity of light.
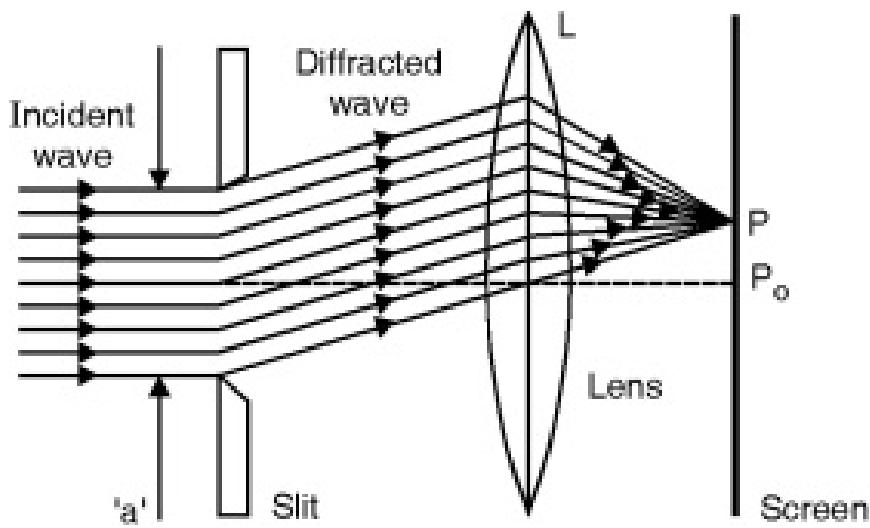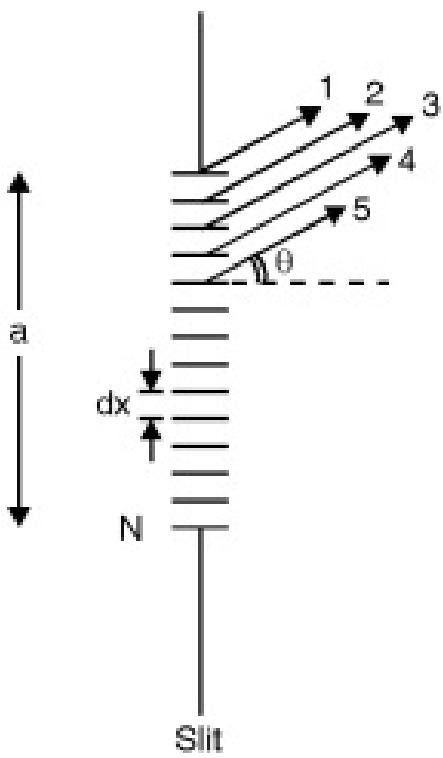
Fig. 3.4.1 : Fraunhoffer Diffraction at Single Slit

- Consider a point P on the screen at which the secondary wavelets travelling at an angle θ with the normal are focussed.

- The intensity of light at point P will depend upon the path difference between the secondary wavelets originating from the corresponding points of the wavefront.
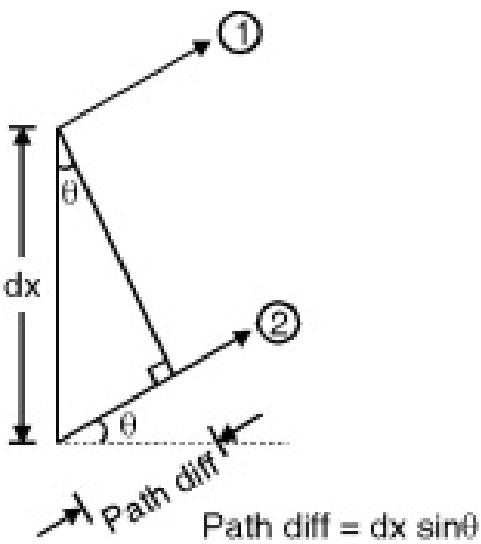
Step II

- Consider the given slit to be divided into N parallel slits, each of width dx. See Fig. 3.4.2(a).

$$\therefore \quad a = dx_1 + dx_2 + dx_3 + \ldots + dx_N \qquad \ldots(3.4.1)$$

- One of such slits is shown in Fig. 3.4.2(b).

(a)

Path diff = dx sinθ

(b)

Fig. 3.4.2

- The path difference between the rays diffracted from its upper and lower edges is dx · sin θ and hence the corresponding phase difference between them is, (refer Fig. 3.4.2(b)).

$$\Delta\phi = \frac{2\pi}{\lambda} dx \sin\theta \qquad\qquad ...(3.4.2)$$

because

Path difference	Phase difference
λ	2π
$dx\sin\theta$	$\Delta\phi$

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda} dx\sin\theta$$

- The total path difference between the rays diffracted from the top and bottom edges of the slit of width 'a' is asinθ and the corresponding phase difference between them is,

$$\phi = \frac{2\pi}{\lambda} \cdot a \cdot \sin\theta \qquad\qquad ...(3.4.3)$$

- Each infinitesimal slit acts as a radiator of Huygen's wavelets and produces a characteristic wave disturbance at point P on the screen.

   This disturbance we represent by a quantity called as phasor. (Concept of phasor is useful when we are dealing in that area where not only the magnitude but also the phase difference is significant.)

- To find out the total amplitude of disturbance at point P, we have to consider N phasors corresponding to N parallel infinitesimal slits.

- Thus at point P, N phasors with same amplitude, same frequency and same phase difference $\Delta\phi$ between the adjacent members combine to produce the resultant disturbance. Fig. 3.4.2 shows the vector addition of such N phasors.

- The resultant disturbance at P is represented by the arc AB of a circle with radius R.

- Let C be the centre of arc AB.

    $\therefore$ $AC = BC = R$

Step III

- Join the chord AB and draw CD perpendicular to AB.
- ϕ is the phase difference between the infinitesimal vectors at the ends of the arc AB i.e. it is the phase difference between the rays coming from the top and bottom edges of the slit of width 'a'.

    $\therefore$ $\angle ACB = \phi$

- Let us represent arc $AB = E_m$ and chord $AB = E_\theta$

    where $E_m \rightarrow$ Amplitude at the centre and $E_\theta \rightarrow$ resultant amplitude at P.

- From the geometry of Fig. 3.4.3, we have,

    $AD = DB$
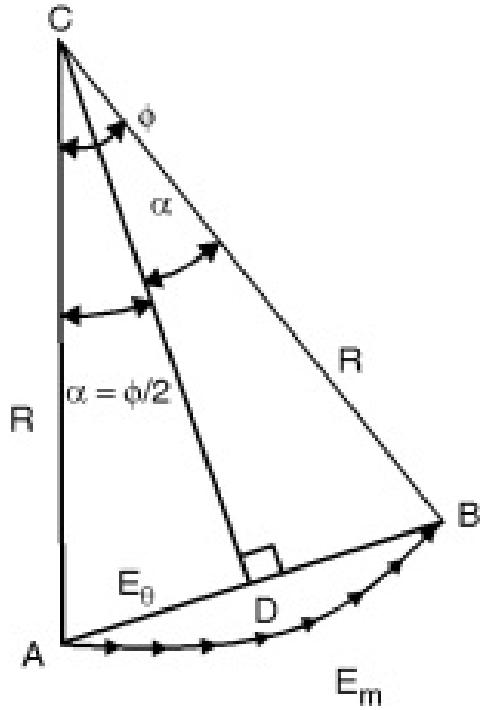
Fig. 3.4.3 : Phasor diagram

$$\angle\, ACD = \angle\, BCD$$

$$= \frac{\phi}{2} = \frac{\pi}{\lambda} \cdot a \sin \theta = \alpha \qquad \text{...(3.4.4)}$$

$$\therefore \quad AD = \frac{E_\theta}{2} = R \cdot \sin \frac{\phi}{2}$$

$$\therefore \quad E_\theta = 2R \cdot \sin \frac{\phi}{2} \qquad \text{...(3.4.5)}$$

Using the relation, angle $= \dfrac{\text{arc}}{\text{radius}}$, we have,

$$\phi = \frac{E_m}{R} \qquad \therefore R = \frac{E_m}{\phi}$$

Therefore Equation (3.4.5) gives,

$$E_\theta = 2 \cdot \frac{E_m}{\phi} \cdot \sin \frac{\phi}{2} = E_m \frac{\sin \phi/2}{\phi/2}$$

$$\therefore \quad \boxed{E_\theta = E_m \cdot \frac{\sin \alpha}{\alpha}} \qquad \text{...(3.4.6)}$$

- This gives the amplitude of wave disturbance at point P where the rays diffracted at angle θ meet.

- If $I_\theta \rightarrow$ Resultant intensity of light at P and $I_m \rightarrow$ Maximum intensity at P_0 then we have $I_\theta \propto E_\theta^2$ and $I_m \propto E_m^2$

- Hence squaring Equation (3.4.6) gives,

$$I_\theta = I_m \cdot \left(\frac{\sin \alpha}{\alpha}\right)^2 \qquad \text{...(3.4.7)}$$

Step IV : Principal Maximum

- For E_θ to be maximum; all the phasors must be in phase i.e. α i.e. $\alpha = 0$

$$\therefore \qquad \alpha = \frac{\pi}{\lambda} \cdot a \sin \theta = 0$$

$$\Rightarrow \qquad \sin \theta = 0 \ \text{ or } \ \theta = 0$$

- Thus the maximum value of E_θ is E_m and this principal maximum is formed at $\theta = 0$. i.e. this maximum is formed by the parts of the secondary wavelets which travel normally to the slit. Hence it is produced at P_0.

Step V : Minimum Intensity

- The intensity at point P will be zero when $\sin \alpha = 0$.
- The values of α which satisfy this condition are

$$\alpha = \pm \, m \, \pi \qquad \text{where} \qquad m = 1, 2, 3, 4, \dots..$$

- Thus the condition for minimum intensity at point P is

$$\alpha = \frac{\pi}{\lambda} \cdot a \sin \theta = \pm m\pi$$

  or $\qquad a \sin \theta = \pm m\lambda \qquad$ ...(3.4.8)

  where $\qquad m = 1, 2, 3, \ldots$

  ($m = 0$ is not possible because then θ becomes zero which corresponds to principal maximum).

- Thus Equation (3.4.8) shows that we have points of minimum intensity on either side of the principal maximum (as there is $\pm$ sign on RHS).

Step VI : Secondary Maxima

- In addition to the principal maximum at $\theta = 0$, there are weak secondary maxima on either side of it. They lie approximately halfway between the two minima.

- Hence the secondary maxima can be obtained for

$$\alpha = \pm \left(m + \frac{1}{2}\right)\pi \quad \text{where} \quad m = 1, 2, 3, \ldots$$

- Putting this condition in Equation (3.4.7) we have the relative intensity of secondary maxima as

$$\frac{I_\theta}{I_m} = \left[\frac{\sin\left(m + \frac{1}{2}\right)\pi}{\left(m + \frac{1}{2}\right)\pi}\right]^2$$

  Putting $m = 1, 2, 3$, we have

$$\frac{I_\theta}{I_m} = 0.045; \ 0.016; \ 0.008; \ \ldots$$

- Thus the successive maxima decrease in intensity rapidly.

- The relative intensity distribution in single slit diffraction pattern is shown in Fig. 3.4.4.
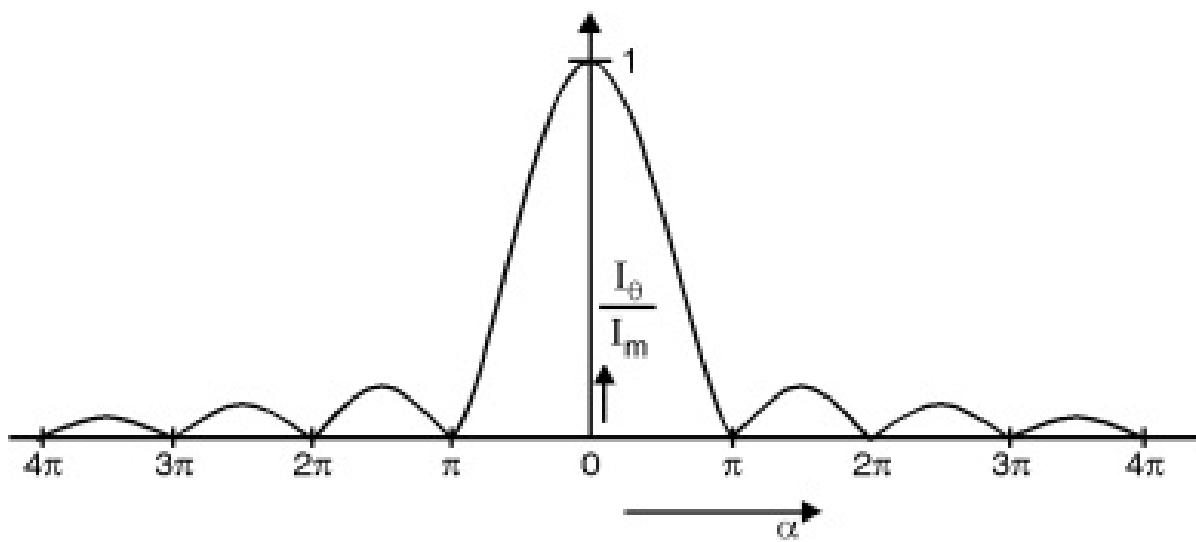
Fig. 3.4.4 : Relative intensity distribution in single slit diffraction

Syllabus Topic : Fraunhoffer Diffraction at a Double Slit

Q.   Explain vector addition method.

Ans. :

Vector addition method

- The resultant amplitude at P can be obtained from Fig. 3.5.2.

- QR and RS represent amplitude of two waves originating from S_1 and S_2 and angle ϕ as phase difference between them.

- From Fig. 3.5.2, we can write,
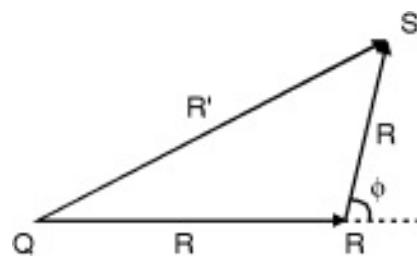
Fig. 3.5.2

$$QS^2 = QR^2 + RS^2 + 2\,(QR)\,(RS)\cos\phi$$

$$R'^2 = R^2 + R^2 + 2R \cdot R \cdot \cos\phi = 2R^2\,(1 + \cos\phi)$$

$$R'^2 = 4R^2 \cos^2 \phi/2$$

Substitute the following values from theory of single slit

$$R = A\frac{\sin\alpha}{\alpha} \text{ and}$$

$$\alpha = \frac{\phi}{2}$$

$$\frac{\pi\,(a + b)\sin\theta}{\lambda} = \beta$$

Hence, $$R'^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Hence, the resultant intensity at the point P is given by,

$$I_\theta = 4E_m^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta \qquad\qquad ...(3.5.4)$$

$$= I_m\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Where, $A^2 = 4\,E_m^2 = \text{constant} = I_m$

- The term $\dfrac{I_m \cdot \sin^2\alpha}{\alpha^2}$ gives diffraction pattern like that of a single slit. **Hence it is called diffraction componment.** The factor $\cos^2\beta$ gives the interference pattern due to light waves of same amplitude from the two slits. **Hence it is called interference component.**

Q.   Explain the effect of increasing the number of slits.

Ans. :

Effect of increasing the number of slits

- Instead of two slits, if we increase the number of slits to 3, 5, 6, ... to a large value, each slit being of same width 'a' and for equal separation 'b' between them, the diffraction patterns in each case can be obtained.

- It is found that with the increase in the number of slits, the narrowing of interference maxima takes place.

- With more number of slits, the sharpness of the principal maxima increases.

- Effect of increasing the slit separation 'b' : Keeping the slit width a constant, if the slit separation b is increased, then the fringe spacing decreases and the fringes become closer together. Hence, more interference maxima lies within the central maximum.

- Effect of increasing the wavelength 'λ' : If the wavelength of the monochromatic light incident on the slit increases, the field of view becomes broader and the fringes move further apart.

- Effect of increasing the slit width 'a' : When the width of the slit 'a' is increased, the envelope of the fringe-pattern changes and central peak becomes sharper. The fringe spacing remains unaffected because it depends on slit separation 'b', which is constant in this case. So, the number of interference maxima lies within the central diffraction maximum decreases.

Q.   Explain the missing order in double slit diffracton.

Ans. :

Missing Orders in a Double Slit Diffraction Pattern

- On keeping the slit width 'a' constant, if the slit spacing 'b' is changed, the fringe width of interference maxima changes. On varying b, some orders of interference maxima are missing. The missing orders in double slit diffraction pattern depend upon the relative values of a and b.

- The direction of interference maxima is given by,

$$(a + b) \sin \theta = n\lambda \qquad \ldots(3.6.1)$$

And the direction of diffraction minima is given by,

$$a \sin \theta = m\lambda \qquad \ldots(3.6.2)$$

- For the same angle of θ, the value of a and b satisfies the Equations (3.6.2) and (3.6.1) simultaneously. Then the position of interference maxima and diffraction minima are same.

- Dividing Equation (3.6.1) by Equation (3.6.2), we get,

$$\frac{(a + b) \sin \theta}{a \sin \theta} = \frac{n\lambda}{m\lambda} \qquad \ldots(3.6.3)$$

$$\frac{a + b}{a} = \frac{n}{m} \qquad \ldots(3.6.4)$$

Condition 1 : If a = b, then,

$$\frac{n}{m} = 2$$

n = 2m

If m = 1, 2, 3, …, then n = 2, 4, 6, …

So, second, fourth, sixth orders of interference maxima are missing in diffraction pattern, because these maxima will coincide with 1st, 2nd, 3rd … order diffraction minima due to single slit.

Condition 2 : If 2a = b, then,

$$\frac{n}{m} = 3$$

n = 3m

If m = 1, 2, 3, … then, n = 3, 6, 9, …

So, 3rd, 6th, 9th orders of interference maxima are missing in the diffraction pattern, because these maxima

will coincide with 1st, 2nd, 3rd … order of diffraction minima.

Condition 3 : If a + b = a, i.e. b = 0

The two slits are joined and all orders of interference maxima are missing and the diffraction pattern obtained is similar to that of single slit of width 2a.

Syllabus Topic : Diffraction Grating

Q.   Explain how the number of lines on grating decides the maximum number of orders of diffraction ? (May 2012)

OR

Q.   What is grating element ? Derive condition for maximum diffraction at diffraction grating. (May 2014)

OR

Q.   What is grating and grating element ? (Dec. 2014, Dec. 2016)

OR

Q.   For plane transmission grating prove that $d \sin \theta = n\lambda$, $n = 1, 2, 3\ldots$ (Dec. 2014)

Ans. :

Grating

It is an arrangement consisting of a large number of parallel slits of same width and separated by equal opaque spaces, is called grating.

Theory of plane transmission grating

Step I

- Consider a plane transmission grating placed perpendicular to the plane of the paper as shown in Fig. 3.7.1.

- Let N be the number of parallel slits each of width 'a' and separated by opaque space 'b'.

- Then the distance between the centres of the adjacent slits is d = (a + b) and is known as the grating element.

- Let a plane wavefront of monochromatic light of wavelength λ be incident normally on the grating.

- When the incident wavefront is at the plane of the slits, every point in each slit acts as a source of secondary wavelets which spread out in all directions.

- The secondary wavelets travelling in the same direction as that of the incident light are brought to focus at point P_0 on the screen by the lens L.
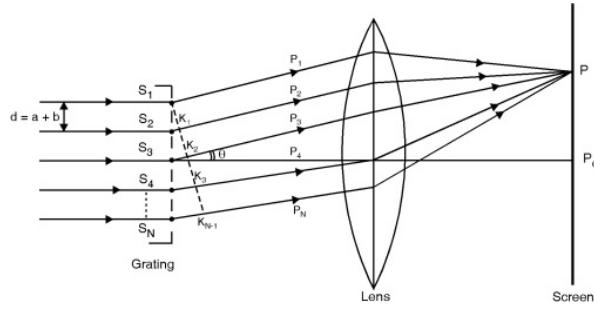
Fig. 3.7.1 : Plane Diffraction Grating

- The screen is placed at the focal plane of the lens.

- The secondary waves travelling in the direction θ with the incident light are focussed at point P on the screen.

- We have to find the resultant intensity of light at P.

Step II

- We have seen that the resultant amplitude of light from a single slit of width 'a' in a direction making an angle θ with the normal is given by,

$$E_\theta = E_m \cdot \frac{\sin \alpha}{\alpha} \qquad ...(3.7.1)$$

$$\text{Where} \quad \alpha = \frac{\pi}{\lambda} \cdot a \sin \theta \qquad ...(3.7.2)$$

$$E_m \rightarrow \qquad \text{Maximum amplitude}$$

- All the secondary wavelets in each slit can be replaced by a single wave of amplitude $E_m \cdot \frac{\sin \alpha}{\alpha}$ starting from the mid - point of the slit and travelling at an angle θ with the normal.

- Let S_1, S_2, S_3, ...S_N be the midpoints of the N number of slits in the grating.

- We want to find the resultant effect of these N vibrations at P.

Step III

- From S_1, draw S_1K_{N-1}, perpendicular on the parallel paths of the diffracted rays.

- Then the path difference between S_1P_1 and S_2P_2 is given by,

  $S_2K_1 = (a + b) \sin \theta$

  and the corresponding phase difference is,

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot (a + b) \sin \theta$$

- Similarly, the path difference between S_3P_3 and S_1P_1 is given by,

  $S_3K_2 = 2 (a + b) \sin \theta$

and the corresponding phase difference is,

$$\frac{2\pi}{\lambda} \cdot 2\,(a+b)\sin\theta = 2\Delta\phi$$

- Thus the phase difference between successive vibrations is

$$\Delta\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta$$

- Hence to find the resultant amplitude at P, we have to find the resultant of N vibrations of equal amplitude $E_m \cdot \frac{\sin\alpha}{\alpha}$, equal periods; but the phase difference between the adjacent vibrations being constant and equal to

$$\Delta\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta \text{ say} \qquad \qquad ...(3.7.3)$$

Step IV

- The resultant amplitude of these N waves can be found out by vector addition method and is given by

$$E_\theta = E_m \frac{\sin\alpha}{\alpha} \frac{\sin N\beta}{\sin\beta} \qquad \qquad ...(3.7.4)$$

- The corresponding resultant intensity of light at P is given by,

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2\beta} \qquad \qquad ...(3.7.5)$$

- The first factor $I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$ in Equation (3.7.5) gives the intensity distribution in the diffraction pattern due to single slit.

- The second factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives intensity pattern due to N slits.

- Reader can try this take N = 1

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$$

- This is the formula for single slit Equation (3.4.7)

For N = 2

$$I_\theta = 4E_n^2 \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \beta$$

(As $\sin 2\beta = 2\cos\beta.\sin\beta$)

This the formula for double slit (Refer Equation (3.4.7))

- Thus every slit gives rise to a diffracted beam whose intensity depends upon the slit width and these diffracted beams then interfere to produce the final diffraction pattern. This is shown in Fig. 3.7.2.
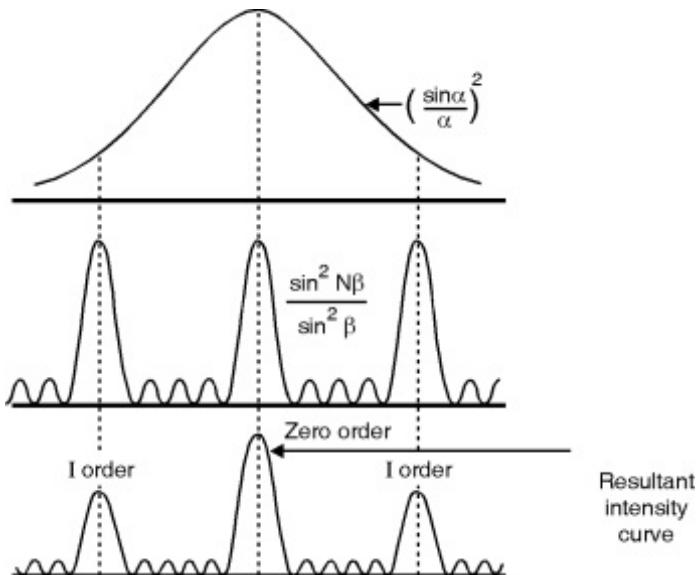
Fig. 3.7.2

Step V : Principal maxima

- The resultant intensity of light at P is given by Equation (3.7.5).

- The intensity is maximum when

$$\sin \beta = 0$$

or $\qquad \beta = \pm m\pi \qquad$ where m = 0, 1, 2, 3, ...

- But for these values of β, the term $\dfrac{\sin N\beta}{\sin\beta}$ in Equation (3.7.5) becomes indeterminate. We find its value as,

$$\lim_{\beta \to \pm m\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm m\pi} \frac{d(\sin N\beta)}{d(\sin\beta)} = \lim_{\beta \to \pm m\pi} \frac{N\cos N\beta}{\cos\beta}$$

$$= \frac{N\cos N\cdot m\pi}{\cos m\pi} = N$$

$$\therefore \lim_{\beta \to \pm m\pi} \left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$$

- Thus for $\beta = \pm m\pi$ the intensity of the principal maxima is,

$$I_\theta = N^2 \cdot I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{which increases with increasing } N.$$

- The intensity of the central principal maximum is greatest while on either side of it, the intensities of other maxima go on decreasing.

- Thus the condition for principal maxima is $\beta = \pm m\pi$

$$\text{or} \quad \frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$\text{or} \quad (a+b)\sin\theta = \pm m\lambda \ldots \qquad \ldots(3.7.6)$$

where $m = 0, 1, 2, 3,$ …..m is called order number.

m = 0 corresponds to zero order maximum i.e. the central maximum which occurs at P_0 i.e. in a direction $\theta = 0$.

- For m = 1, 2, 3, ... we get the first order. Second order, ... principal maxima respectively.

- The ± sign in Equation (3.7.6) shows that there are two principal maxima of same order lying on either side of the central principal maximum.

- As number of lines on grating increases, (a + b) decreases. Because $a + b = \frac{1}{\text{Number of lines per inch}}$. Also (a + b) sin θ = mλ.

$\therefore$ With increase in number of lines on grating order decreases (For given θ and λ).

Step VI : Minima

- From Equation (3.7.5) it is seen that the intensity of light at P is minimum when

$$\sin N\beta = 0 \qquad \text{but } \sin \beta \neq 0.$$

$$N\beta = \pm m\pi$$

where m can have all integral values except 0, N, 2N, ...nN; because for these values of m, sin β becomes zero and we get principal maxima.

- Hence putting for β from Equation (3.7.3) we have the condition for minima as

$$N\frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

or $N (a + b) \sin \theta = \pm m\lambda$ .....(3.7.7)

where m can have all integral values except 0, N, 2N, ...nN.

Q.   Derive condition for absent spectra in grating. (May 2013, Dec. 2016)

Ans. :

Condition for Absent Spectra

- The intensity of light due to a diffraction grating in a direction making an angle θ with the normal to the grating is given by,

$$I_\theta = I_m \left(\frac{\sin^2 \alpha}{\alpha^2}\right)\left(\frac{\sin^2 N\beta}{\sin^2 \beta}\right)$$

where $\alpha = \frac{\pi}{\lambda} a \sin \theta$

and $\beta = \frac{\pi}{\lambda}(a + b) \sin \theta$

- The factor $I_m \dfrac{\sin^2 \alpha}{\alpha^2}$ gives the intensity distribution due to a single slit while the factor $\dfrac{\sin^2 N\beta}{\sin^2 \beta}$ gives that due to the combined effect or N slits of the grating.

- Now the principal maxima in case of a grating are obtained in the directions given by,

$$(a + b) \sin \theta = m\lambda \qquad \qquad ...(3.8.1)$$

  m being the order of the spectrum.

- Also the minima in case of a single slit are obtained in the directions given by,

$$a \sin \theta = n\lambda \qquad \qquad ...(3.8.2)$$

$$n = 1, 2, 3, ...$$

- If both the conditions (3.8.1) and (3.8.2) are satisfied simultaneously, a particular maximum of order m will be missing in the grating spectrum. Hence dividing (3.8.1) by (3.8.2) we have

$$\frac{a + b}{a} = \frac{m}{n} \qquad \qquad ...(3.8.3)$$

  which is the condition for absent spectra.

- If a = b, then we have from Equation (3.8.3).

$$\frac{m}{n} = 2$$

  or   m = 2 n   where   n = 1, 2, 3, ...

  i.e. 2nd, 4th, 6th, ... etc. order spectra will be absent.

  i.e. the even order spectra will be absent if the width of ruling is equal to width of the slit.

Syllabus Topic : Determination of Wavelength of Light with Plane Transmission Grating

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Q.   Explain the experimental method to determine the wavelength of spectral line using diffraction grating. (Dec. 2012, May 2016)

Ans. :

Determination of Wavelength of Light using Diffraction Grating

- The diffraction grating is often used in the laboratories for the determination of wavelength of light.

- The grating spectrum of the given source of monochromatic light is obtained by using a spectrometer. The arrangement is as shown in Fig. 3.10.1.

- The spectrometer is first adjusted for parallel rays. The grating is then placed on the prism table and adjusted for normal incidence.

- In the same direction as that of the incident light, the direct image of the slit or the zero order spectrum can be seen in the telescope.
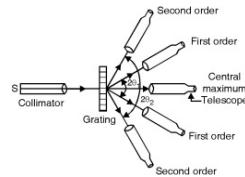
Fig 3.10.1

- On either side of this direct image a symmetrical diffraction pattern consisting of different orders can be seen.

- The angle of diffraction θ for a particular order m of the spectrum is measured.

- The numbers of lines per inch of grating are written over it by the manufacturers.

- Hence the grating element is,

$$(a+b) = \frac{1}{\text{Number of lines/cm}} = \frac{2.54}{\text{Number of lines/inch}}$$

- Thus using the equation

$$(a + b) \sin \theta = m\lambda$$

- The unknown wavelength λ can be calculated by putting the values of the grating element $(a + b)$, the order m and the angle of diffraction θ .

Syllabus Topic : Dispersive Power of a Grating

Q.   What is Rayleigh's criteria of resolution ? (May 2014, May 2015, Dec. 2016)

Ans. :

Rayleigh's Criterion of Resolution

- According to Rayleigh's criterion, two closely spaced point sources of light are said to be just resolved by an optical instrument only if the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction pattern of the other and vice versa.

- In order to illustrate the criterion consider the resolution of two wavelengths λ_1 and λ_2 by an optical instrument. Fig. 3.13.1 shows the intensity curves of the diffraction patterns of the two wavelengths.

- As shown in Fig. 3.13.1(a) the principal maxima of the two wavelengths are widely separated. Hence the two wavelengths are well resolved.
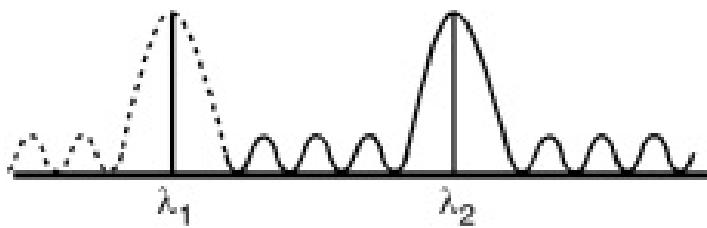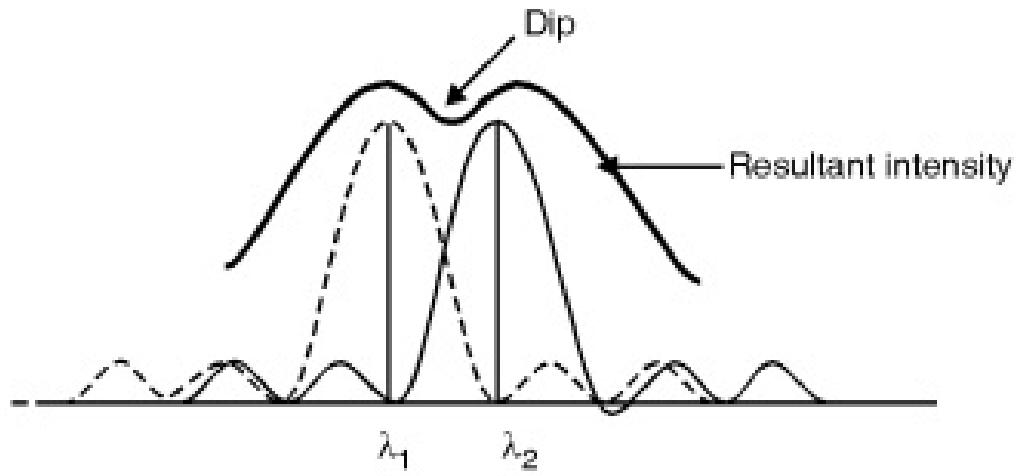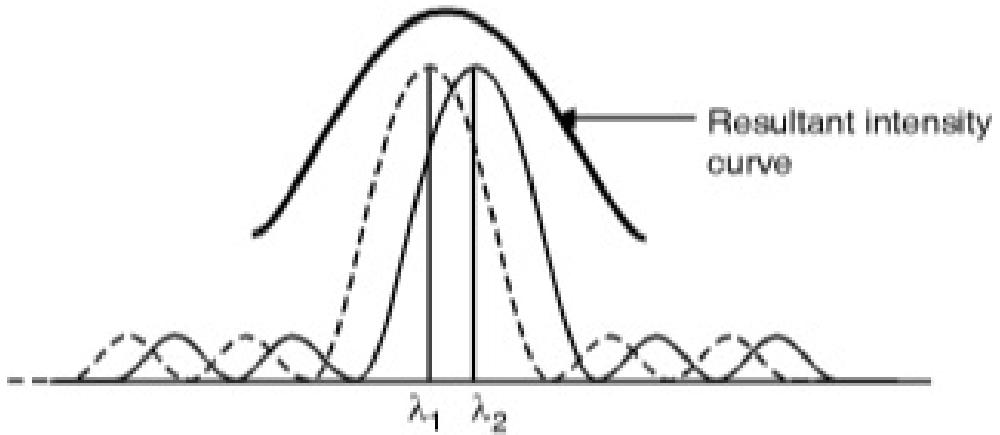
Fig. 3.13.1(a) : Objects Well Resolved

- Fig. 3.13.1(b) shows the intensity curves of two wavelengths whose difference is such that the principal maximum of one coincides with the first minimum of the other and vice versa.

- The resultant intensity curve shows a distinct dip in the middle indicating the presence of two wavelengths. This is the limiting case when the two wavelengths are just resolved.

Dip

Resultant intensity

λ_1 λ_2

(b) Objects Just Resolved

Resultant intensity curve

λ_1 λ_2

(c) Objects not Resolved

Fig. 3.13.1

- It however; the two wavelengths are still closer, their principal maxima will be still nearer as shown in Fig.

3.13.1(c).

- The resultant intensity curve shows only one maximum at the centre. The two wavelengths cannot be seen as separate and hence are not resolved.

- Thus the spectral lines can be resolved only upto the limit expressed Rayleigh's Criterion.

<div style="background-color: #d4edda; text-align: center; font-weight: bold; border-top: 3px solid black; border-bottom: 3px solid black; padding: 8px;">Syllabus Topic : Resolving Power of a Grating</div>

Q.   How to increase resolving power of diffraction grating ? (Dec. 2013, Dec. 2016)

Ans. :

of diffraction grating

Step I

- Consider a grating on which light consisting of two close wavelengths λ and $(\lambda + d\lambda)$ is incident normally.

- Let $(a + b) \rightarrow$ grating element and $N \rightarrow$ Total number of lines on the grating.

- These two wavelengths will be just resolved if the principal maximum of $(\lambda + d\lambda)$ falls over the first minimum of λ.

- As per the Rayleigh's criterion, the first minimum of λ adjacent of mth principal maximum should be along $(\theta_m + d\theta_m)$ for just resolution.
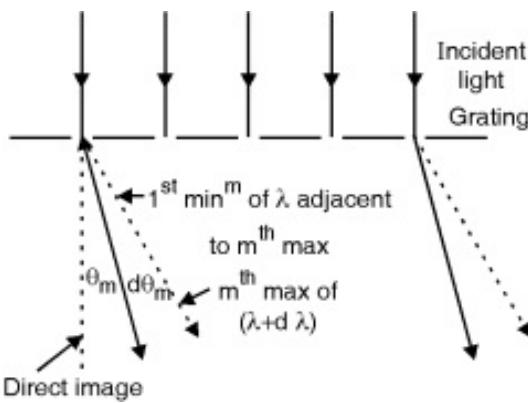
Fig. 3.14.1

Step II

- The mth principal maximum of $(\lambda + d\lambda)$ along $(\theta_m + d\,\theta_m)$ is given by the equation.

$$(a + b) \sin (\theta_m + d\,\theta_m) = m\,(\lambda + d\lambda) \qquad ...(3.14.1)$$

- The equation for minima for λ is

$$N\,(a + b) \sin \theta_m = m\lambda \qquad .....(3.14.2)$$

Where, m has all integral values except 0, N, 2N, 3N, ...nN.

- Thus the first minimum of λ adjacent to m^{th} principal maximum can occur along $(\theta_m + d\,\theta_m)$ by putting m as (m N +1) in Equation (3.14.2).

- Hence the equation for first minimum of λ in the direction $(\theta_m + d\,\theta_m)$ will be given by,

$$N\,(a + b) \sin (\theta_m + d\,\theta_m) = (m.N + 1).\lambda \qquad ...(3.14.3)$$

Step III

- Multiply Equation (3.14.1) by N

$$\therefore\ N(a + b)\sin(\theta_m + d\,\theta_m) = mN(\lambda + d\lambda)$$

$$...(3.14.4)$$

- Therefore from Equations (3.14.3) and (3.14.4) we have,

$$(m \cdot N + 1) \cdot \lambda = mN(\lambda + d\lambda)$$

$$\therefore\ \lambda = m\,N \cdot d\lambda$$

$$\therefore\ \frac{\lambda}{d\lambda} = m \cdot N$$

$$\therefore\ \text{R.P. of grating} = \frac{\lambda}{d\lambda} = m \cdot N \qquad .....(3.14.5)$$

Thus from Equation (3.14.5) we see that :

(i)   The R.P. of grating increases with the order of spectrum

(ii) The R.P. increases with the increase in total number of lines on the grating.

(iii) The R.P. is independent of the grating element.

Q.   What is resolving power of diffraction grating ? (May 2014, May 2015, Dec. 2016)

Ans. :

Resolving power of diffraction grating

The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in the wavelength between this line and a neighbouring line such that the two lines appear to be just resolved.

$$\text{R.P} = \frac{\lambda}{d\lambda}$$

Where, $\lambda \rightarrow$ wavelength of a line

and $\lambda + d\lambda \rightarrow$ wavelength of the next line that can just be seen as separate.

Solved Problems

Ex. 3.14.1 (Dec. 2014, 5 Marks)

Calculate the minimum number of lines in a grating which will just resolve in the first order the lines whose wavelengths are 5890 Å and 5896 Å.

Soln. :

Given : m = 1, λ_1 = 5890 Å, λ_2 = 5896 Å

$\therefore$ dλ = $\lambda_2 - \lambda_1$ = 6 Å

$\therefore$ Mean wavelength λ = 5893 Å

Formula : $\quad\quad\quad\quad$ R.P. $= mN = \dfrac{\lambda}{d\lambda}$

$$N = \frac{\lambda}{d\lambda} \cdot \frac{1}{n} = \frac{5893 \times 10^{-10}}{6 \times 10^{-10}} \times \frac{1}{1}$$

$\quad\quad$ = 983 $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ ...Ans.

Ex. 3.15.5 (May 2015, 5 Marks)

A slit of width 0.3 mm is illuminated by a light of wavelength 5890 Å. A lens whose focal length is 40 cm forms a Fraunhofer diffraction pattern. Calculate the distance, between first dark and the next bright fringe form the axis.

Soln. :

Given : $\quad\quad\quad\quad\quad$ a = 0.3 mm

$\quad\quad$ λ = 5890 A°

$\quad\quad$ f = 40 cm

To find : $x_2 - x_1$

$\quad\quad$ where x_1 = Distance between centre of fringe pattern and the first order minima

$\quad\quad\quad$ x_2 = Distance between centre of fringe pattern and next bright maxima.

$\quad$ Use the following diagram.
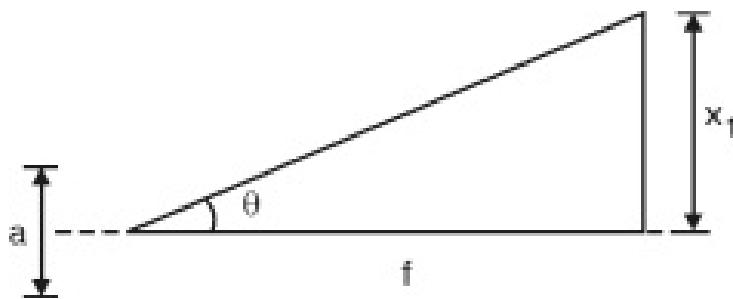
Fig. P. 3.15.5

As θ is very small, $\sin \theta \simeq \theta = \dfrac{x_1}{f}$

Condition for minima in single slit.

$a \sin \theta = m\lambda$

$$4a\left(\dfrac{x_1}{f}\right) = m\lambda$$

$$\dfrac{x_1}{f} = \dfrac{\lambda}{a} \qquad \text{(for } m = 1)$$

$$4 \quad x_1 = \dfrac{f \times \lambda}{a} = \dfrac{5890 \times 10^{-8} \times 40}{0.3 \times 10^{-1}} = 0.0785 \text{ cm} \qquad ...(1)$$

Similarly for maxima in single slit

$$a \sin \theta = \left(m + \dfrac{1}{2}\right)\lambda = \dfrac{3}{2}\lambda \qquad \text{(for } m = 1)$$

Consider above mentioned diagram for maxima

we get $\sin \theta \simeq \theta = \dfrac{x_2}{f}$

$$4a \times \dfrac{x_2}{f} = \dfrac{3}{2}\lambda$$

$$4 \dfrac{x_2}{f} = \dfrac{3}{2}\dfrac{\lambda}{a}$$

$$x_2 = \dfrac{3}{2} \times \dfrac{5890 \times 10^{-8} \times 40}{0.03} = 0.1178 \text{ cm} \qquad ...(2)$$

∴ Dist between the first dark and next bright fringes

$$x_2 - x_1 = 0.0393 \text{ cm} \qquad \text{...Ans.}$$

Ex. 3.15.12 (May 2013, 3 Marks, May 2014, 5 Marks)

In a plane transmission grating the angle of diffraction for second order principal maximum for the wavelength 5×10^{-5} cm is 30°. Calculate the number of lines/cm of the grating surface.

Soln. :

Given : $\qquad\qquad\qquad\qquad\qquad \lambda = 5 \times 10^{-5}$ cm, $\theta = 30°$, $n = 2$

$$\therefore \quad n\lambda = (a + b) \sin \theta$$

$$\therefore a + b = \frac{n\lambda}{\sin \theta}$$

$$= \frac{2 \times 5 \times 10^{-5}}{\sin (30°)}$$

$$= 2 \times 10^{-4}$$

$$\therefore \text{No. of lines per cm} = \frac{1}{a + b} = 5000 \qquad \text{...Ans.}$$

Ex. 3.15.16 (Dec. 2013, 5 Marks)

What is the highest order spectrum that is visible with light of wavelength 6000 Å by means of a diffraction grating having 5000 lines per cm ?

Soln. :

Given : $\lambda = 6000 \text{ Å} = 6000 \times 10^{-10}$ m $= 6000 \times 10^{-8}$ cm

Number of lines per cm $= 5000$

$$\therefore \text{Grating element } (a + b) = \frac{1}{5000} \text{ cm}$$

The grating equation is $(a + b) \sin \theta = m\lambda$

m is maximum when $\sin \theta = 1$

$$\therefore m_{max} = \frac{a + b}{\lambda} = \frac{1}{5000} \times \frac{1}{6000 \times 10^{-8}}$$

$$= 3.33$$

$$\therefore \text{Maximum order of spectrum} = 3 \qquad \text{...Ans.}$$

Ex. 3.15.18 (May 2015, 5 Marks)

Calculate the maximum order of diffraction maxima seen from a plane diffraction grating having 5500 lines per cm if light of wavelength 5896 Å falls normally on it.

Soln. :

Given :

Number of lines per cm = 5500

$\lambda = 5896$ A°

$$4 \quad a + b = \frac{1}{5500}$$

For maximum order take $\sin \theta = 1$ in

$n\lambda = (a + b) \sin \theta$

$$4 \quad n = \frac{a + b}{\lambda} \times \frac{1}{\sin \theta}$$

$$= \frac{1}{5500} \times \frac{1}{5896 \times 10^{-8}} \times \frac{1}{1} = 3 \qquad \text{...Ans.}$$

Ex. 3.15.21 (Dec. 2015, 5 Marks)

The visible spectrum ranges from 4000A° to 7000A°. Find the angular breadth of the first order visible spectrum produced by a plane waling having 6000 lines/cm when light is incident normally on the grating.

Soln. :

Given : Range of wavelength 4000A° to 7000A°

Number of lines / cm = 6000

$$4 \quad (a + b) = \frac{1}{6000} \qquad \text{...(1)}$$

Formula $n\lambda = (a + b) \sin \theta$

$$4 \quad \theta = \sin^{-1}\left(\frac{n\lambda}{a + b}\right)$$

$\therefore$ For $\lambda_1 = 4000\ \text{A}°$

$\theta_1 = \sin^{-1}(4000 \times 10^{-8} \times 6000)$

$= 13.89°$ ...(2)

For $\lambda_2 = 7000\ \text{A}°$

$\theta_2 = \sin^{-1}(7000 \times 10^{-8} \times 6000)$

$= 24.84°$ ...(3)

$\therefore$ Angular breadth, $15 = 24.84 - 13.89$

$= 10.95°$ ...Ans.

Ex. 3.15.23 (Dec. 2015, May 2016, 5 Marks)

A diffraction grating used at normal incidence gives a yellow line ($\lambda = 6000\ \text{Å}$) in a certain spectral order superimposed on a blue line ($\lambda = 4800\ \text{Å}$) of next higher order. If the angle of diffraction is $\sin^{-1}\left(\dfrac{3}{4}\right)$, calculate the grating element.

Soln. :

We have the direction of principal maxima is

$(a + b)\sin\theta = n\lambda$

For yellow line ($\lambda_1 = \text{Å}$), we can write,

$(a + b)\sin\theta = n\lambda_1$ ...(1)

For blue line ($\lambda_2 = 4800\ \text{Å}$), we can write,

$(a + b)\sin\theta = (n + 1)\lambda_2$ ...(2)

Comparing Equations (1) and (2), we get,

$n\lambda_1 = (n + 1)\lambda_2$

$n = \dfrac{\lambda_2}{\lambda_1 - \lambda_2} = \dfrac{4800\ \text{Å}}{(6000 - 4800)\ \text{Å}} = 4$

Given : $\theta = \sin^{-1}\left(\dfrac{3}{4}\right) \Rightarrow \sin\theta = \left(\dfrac{3}{4}\right)$

Put these values in Equation (1), we get,

$$(a+b) = \frac{n\lambda_1}{\sin\theta} = \frac{4 \times 6 \times 10^{-7}}{3/4} = 3.2 \times 10^{-6} \text{ m} \qquad ...\text{Ans.}$$

Ex. 3.15.26 (Dec. 2016, 5 Marks)

In a plane transmission grating, the angle of diffraction for the second order principal maxima for wavelength 5×10^{-5} cm is 35°. Calculate the number of lines/cm on diffraction grating.

Soln. :

Given :

$\theta = 35°$

$\lambda = 5 \times 10^{-5}$ cm

$n = 2$

To find : No. of lines / cm

Formula

$n\lambda = (a+b) \sin\theta$

$2 \times 5 \times 10^{-5} = (a+b) \sin 35°$

$$\therefore \quad (a+b) = \frac{2 \times 5 \times 10^{-5}}{\sin 35}$$

$$\therefore \text{ number of lines per cm} = \frac{1}{a+b} = \frac{\sin 35}{2 \times 5 \times 10^{-5}}$$

$$= 5735 \qquad \qquad ...\text{Ans.}$$

Ex. 3.15.31 (May 2013, 5 Marks)

A plane grating just resolves two lines in the second order. Calculate the grating element if $d\lambda = 6$ Å, $\lambda = 6 \times 10^{-5}$ cm and the width of the ruled surface is 2 cm.

Soln. :

Given : $m = 2$, $d\lambda = 6$ Å $= 6 \times 10^{-8}$ cm, $\lambda = 6 \times 10^{-5}$ cm, W $= 2$ cm.

Formula : (i) $R.P. = \dfrac{\lambda}{d\lambda}$,   (ii) $R.P. = mN$

$$R.P. = \dfrac{\lambda}{d\lambda} = mN$$

$$\therefore \quad N = \dfrac{\lambda}{d\lambda} \cdot \dfrac{1}{m} = \dfrac{6 \times 10^{-5}}{6 \times 10^{-8}} \times \dfrac{1}{2} = 500$$

∴ Number of lines in a width of 2 cm of grating = 500

$$\therefore \text{ Number of lines per cm} = \dfrac{500}{2} = 250$$

$$\therefore \text{ Grating element } (a+b) = \dfrac{1}{\text{Number of lines/cm}}$$

$$\therefore (a+b) = \dfrac{1}{250} = 4 \times 10^{-3} \text{ cm} \qquad \text{...Ans.}$$

Ex. 3.15.34 (Dec. 2014, 5 Marks)

Find the minimum number of lines in a plane diffraction grating required to just resolve the sodium doublet (5890 Å and 5896 Å) in the (i) first order (ii) second order.

Soln. :

We have R.P. of grating,

$$\dfrac{\lambda}{d\lambda} = m \cdot N$$

$$\therefore \quad N = \dfrac{1}{m}\left(\dfrac{\lambda}{d\lambda}\right)$$

(i)   For the first order, m = 1

Average wavelength $\lambda = \dfrac{\lambda_1 + \lambda_2}{2} = \dfrac{5890 + 5896}{2} = 5893$ Å

$d\lambda = 5896 - 5890 = 6$ Å

$N = \frac{5893}{1 \times 6} = 982$ ...Ans.

(ii)   For the second order m = 2, the minimum number of lines,

$N = \frac{5893}{2 \times 6} = 491$ ...Ans.

Ex. 3.15.38 (May 2016, 3 Marks)

A grating has 620 rulings/mm and is 5.05 mm wide. What is the smallest wavelength interval that can be resolved in the third order at $\lambda = 481$ nm ?

Soln. :

$$\text{No. of lines / mm} = 620$$

$$\therefore \quad \text{No. of lines / cm} = N = 6200$$

$$\therefore \quad \text{Order m} = 3$$

$$\lambda = 481 \times 10^{-9} \text{ m} = 418 \times 10^{-7} \text{ cm}$$

$$\frac{1}{d\lambda} = mN$$

$$d\lambda = \frac{\lambda}{mN} = \frac{481 \times 10^{-7}}{3 \times 6200}$$

$$= 2.58 \times 10^{-9} \text{ cm}$$

Smallest wavelength interval = 2.58×10^{-9} cm
...Ans.

$\rightarrow\rightarrow\rightarrow$

Q. What is acronym of 'LASER' ? How are they different than X-rays ? (May 2014)

Ans. :

Acronym of LASER

- LASER is acronym for Light Amplification by Stimulated Emission of Radiation. A laser beam is highly parallel coherent beam of light of very high intensity.

- Production of laser light is a particular consequence of interaction of radiation as a rule with matter.

- The interpretation of the interaction is done on the basis of ideas related to energy levels of the concerned system from which light is derived. In case of gases, it may be noted that, though different gases have different energy level patterns, any gas as a rule will have discrete energy levels, the energy quantization rules always hold good. Hence matter, irrespective of its state of existence, is referred to as a quantized system.

LASER differs from x-rays

Since LASER is high energy beam, at times it is compared with x-rays. But both of them differ completely. Some points at which LASER differs from x-rays are :

(1) LASER is highly coherent where an x-ray is not.

(2) LASER has it wavelength of the order of visible spectrum, where as x-rays has very small wavelength.

(3) Stimulated emission is essential for LASER where as x-ray needs high energy electron and their retardation.

Q. Write short note on : Absorption.

Ans. :

Absorption

- An atom in lower energy state E_1 may absorb the incident photon and may be excited to E_2 as shown in Fig. 4.2.1. This transition is known as stimulated absorption corresponding to each transition made by an atom one photon disappears from the incident beam.

Fig. 4.2.1 : Induced absorption

- The transition may be written as

$$A + h\nu \ = \ A^* \qquad \qquad ...(4.2.1)$$

Where   A = Atom in lower energy state

A* = Atom in excited state

- The number of atoms N_{ab} excited during the time Δt is

given by

$$N_{ab} = B_{12}N_1 Q\Delta t \qquad \ldots(4.2.2)$$

Where $\qquad N_1$ = Number of atoms in state E_1

Q = Energy density of the incident beam

B_{12} = Probability of an absorption transition.

Syllabus Topic : Spontaneous Absorption

Q.   Explain the terms Spontaneous emission.

Ans. :

Spontaneous Emission

- Excited state with higher energy is inherently unstable because of a natural tendency of atoms to seek out lowest energy configuration. Therefore excited atoms do not stay in the excited state for a relatively longer time but tend to return to the lower state by giving up the excess energy $h\nu = E_2 - E_1$ in the form of spontaneous emission or stimulated emission.

- The excited atom in the state E_2 may return to the lower state E_1 on its own out of natural tendency to attain the minimum potential energy condition. During the transition the excess energy is released as a photon of energy $h\nu = E_2 - E_1$. This type of process in which photon emission occurs without any external agency is called spontaneous or natural emission. Fig. 4.2.2 represents natural emission and shows the transition.

$$A^* \rightarrow A + h\nu \qquad \ldots(4.2.3)$$

Fig. 4.2.2 : Natural or Spontaneous emission

- The number of spontaneous transitions N_{sp} taking place in the medium during time Δt depends only on the number of atoms N_2 lying in the excited state E_2. It is given by,

$$N_{sp} = A_{21}N_2\Delta t \qquad ...(4.2.4)$$

  where,

$$N_2 = \text{Number of atoms in the state } E_2$$

$$A_{21} = \text{Probability of a spontaneous emission.}$$

Syllabus Topic : Stimulated Emission

Q.   Explain the term stimulated emission. (May 2016)

Ans. :

Stimulated emission

- An atom in excited state need not wait for spontaneous emission to occur. There exists an additional possibility according to which an excited atom can make a downward transition and emit a radiation.

- A photon of energy $h\nu = E_2 - E_1$ can induce the excited atom to make a downward transition releasing the energy in the form of a photon. Thus the interaction of a photon with an excited atom triggers the excited atom to drop to the lower energy state giving up a photon.

- This phenomenon is called forced emission or stimulated emission as shown in Fig. 4.2.3. The process may be represented as

$$A^* + h\nu = A + 2h\nu \qquad ...(4.2.5)$$
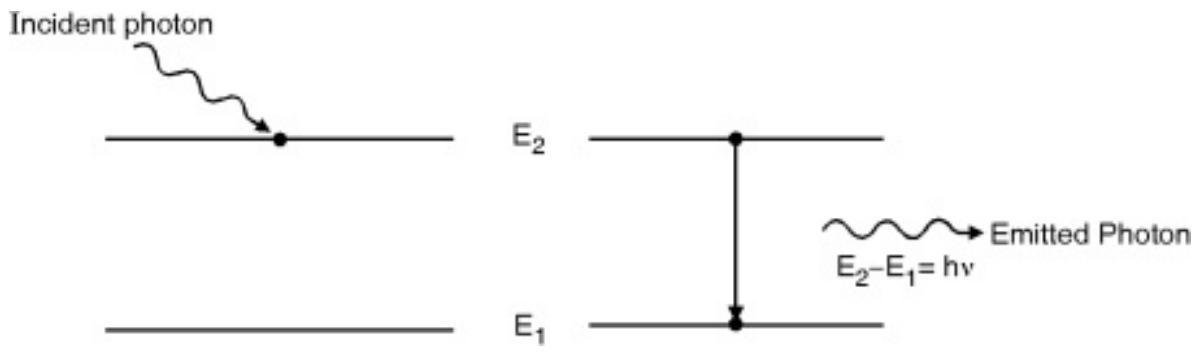
Fig. 4.2.3 : Stimulated emission

Q.   What is the difference between spontaneous and stimulated emission ? (Dec. 2013)

OR

Q.   Differentiate spontaneous and stimulated emission of radiation.   (Dec. 2014, Dec. 2015)

Ans. :

Difference between Spontaneous emission and Stimulated emission

Spontaneous emission	Stimulated emission
1) It is a natural process.	1) Artificial, induced process.
2) Can not be controlled.	2) Can be controlled effectively.
3) No multiplication of photons takes place.	3) Multiplication of photons takes place.
4) Not useful for LASER.	4) Essential for LASER.

Syllabus Topic : Population Inversion

Q.   What is a population inversion state ? Explain its significance in the operation of LASER. (May 2013)

OR

Q.   Explain the term population inversion. (May 2016)

Ans. :

Population inversion

(a) Population of energy level

- For the sake of simplicity, let us consider the atomic state of matter, where we have set of allowed energy levels for an atom. The set of allowed energy levels is the energy level scheme is same for all the atoms of same type.

- If we consider an assembly of identical atoms, then we can compare the energies of all those atoms in a single energy level scheme. Normally, the numbers of atoms are required to possess one or the other energy values which are permitted only in this energy scheme, each of energy states will be having many atoms as its members. The number of atoms in a particular state is referred to as its population.

(b) Boltzmann factor

- The population of different energy states of any physical system are related to each other, provided, the system is in thermal equilibrium. The relation is given by Boltzmann factor. Among the various energy states, if we consider any two energy states E_1 and E_2 with population N_1 and N_2 respectively, and if $E_2 > E_1$, then Boltzmann factor is the ratio (N_2/N_1) given by,

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/KT} \qquad \qquad ...(4.4.1)$$

   Where K is Boltzmann constant.

Since $E_2 > E_1$, $e^{-(E_2 - E_1)/KT} < 1$

∴ $N_2 < N_1$

- Hence for a system in thermodynamic equilibrium, it is mandatory that the population of any higher state is always lesser than that in any of the lower states.

- Population inversion is the state of a system at which the population of a particular energy state is more than that of a specified lower energy state i.e. $N_2 > N_1$. Under normal condition population inversion condition does not exist. However it is possible to achieve the population inversion condition in certain system, which has metastable states.

- It is considered as a precondition of LASER. It makes LASER possible with the help of metastable state.

Syllabus Topic : Metastable States

Q.   Write short note on : Metastable states.

Ans. :

Metastable states

- By providing energy, if an atom is made to go to one of its excited states, it stays there over a brief interval of time not exceeding 10^{-8} sec., and then returns to one of the lower energy states. In case the state to which the atom is excited is a metastable state, then the atom stays there for unusually long time, which is of the order of 10^{-3} to 10^{-2} seconds. This property is essential for achieving population inversion.

- Metastable states may be considered as a special privilege enjoyed by atoms of some specific elements. Such states are not created. Infact, existence of metastable states helps us decide which element is useful for population inversion and which one is not. Like He is useful in He-Ne, Nd is useful in Nd – YAG etc.

Syllabus Topic : Pumping

Q. What is pumping in LASER ? Give the types of pumping. (Dec. 2016)

Ans. :

Pumping

The process of raising large number of atoms from lower energy level to a higher energy level is called pumping.

Types of pumping

- Optical pumping : Which uses strong light source for excitation.
- Electrical pumping : Which uses electron impact for excitation.
- Chemical pumping : Which uses chemical reactions for excitation.
- Direct pumping : Which uses direct conversion of electric energy into light.

Syllabus Topic : Resonance Cavity

Q. Write short note on : Resonance Cavity.

Ans. :

Resonant Cavity

- A laser device consists of an active medium bound between two mirrors or highly reflecting surfaces as shown in Fig. 4.7.1. These surfaces reflect the photons to and fro through the active medium. A photon moving in a particular direction represents a light wave moving in the same direction. Thus the two mirrors along with the active medium form a cavity inside which two types of waves exists, one type comprises of waves moving to the right, and the other one to the left Figs. 4.8.2 and 4.8.3.
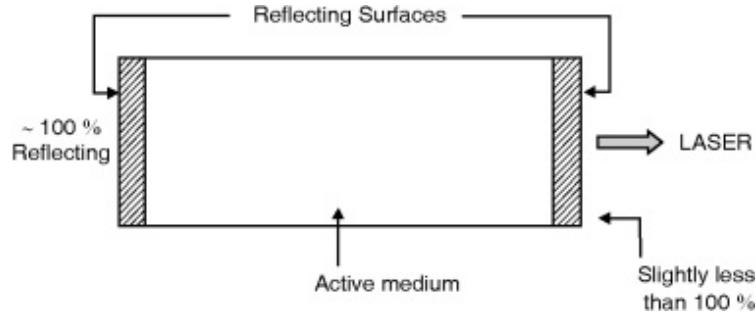
Fig. 4.8.1 : Resonant cavity

- The two waves interfere constructively if there is no phase difference between the two (Fig. 4.8.2) But, their interference becomes destructive if the phase difference is $\pi/2$ (Fig. 4.8.3).
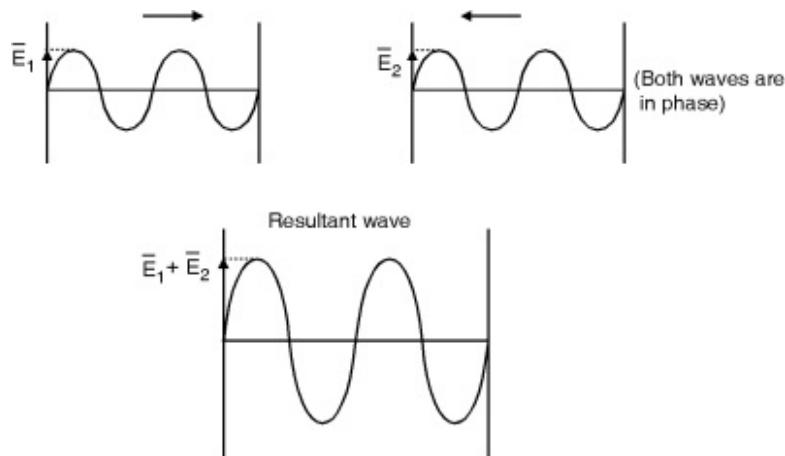
Fig. 4.8.2 : Constructive interference

- For destructive interference.
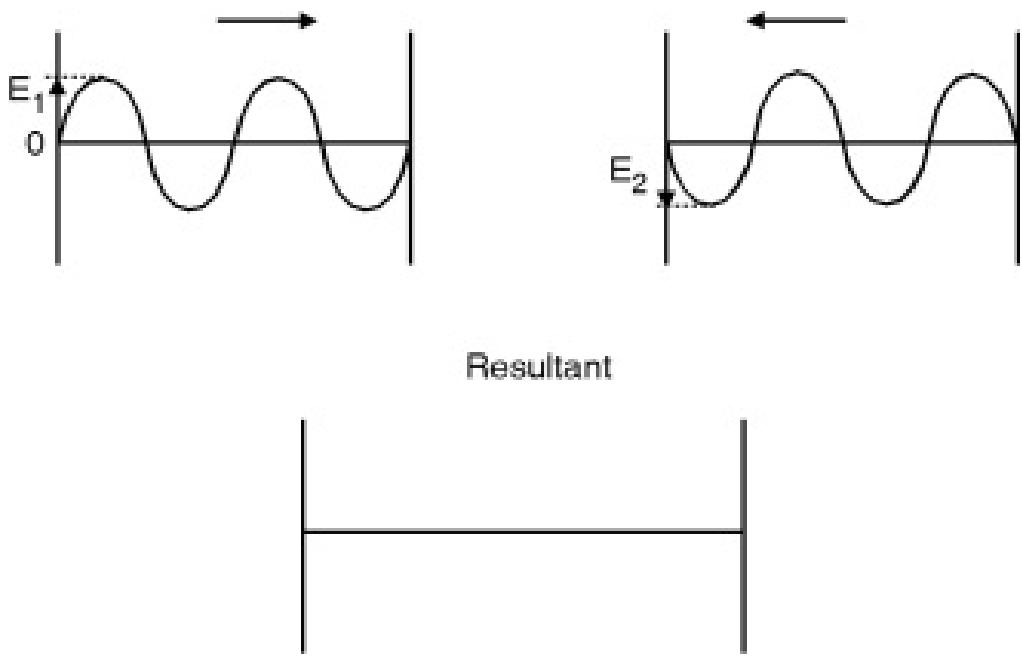
Fig. 4.8.3 : Destructive interference

- In order to arrange for constructive interference, the distance 'L' between the two reflecting surfaces should be such that the cavity should support an integral number of half wavelengths.

$$\therefore \ L \ = \ m \frac{\lambda}{2} \qquad \qquad ...(4.8.1)$$

Where, m is an integer and m > 0, and λ is the wavelength of the laser light.

- In such a case, a standing wave pattern is established within the cavity, and the cavity is said to be resonant at wavelengths.

$$\therefore \ \lambda \ = \ \frac{2L}{m} \qquad \qquad ...(4.8.2)$$

Q. Derive Einstein's relation for stimulated emission.

Ans. :

Einstein's relation for stimulated emission

- Einstein's was the first to calculate the probability of such transition assuming the atomic system to be in equilibrium with electromagnetic radiation.

- Under thermal equilibrium the number of upward transition is equal to number of downward transition per unit volume per second.

$$\therefore \quad A_{21} N_2 + B_{21}N_2Q = B_{12}N_1Q \quad\quad \ldots(4.9.1)$$

- From Equation (4.9.1) we get,

$$Q = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$Q = \frac{\dfrac{A_{21}}{B_{21}}}{\dfrac{B_{12}}{B_{21}}\dfrac{N_1}{N_2} - 1}$$

$$\text{Now} \quad \frac{N_1}{N_2} = \exp\left(\frac{E_2 - E_1}{KT}\right) = \exp\left(\frac{h\nu}{KT}\right)$$

$$\therefore \quad Q = \frac{\dfrac{A_{21}}{B_{21}}}{\dfrac{B_{12}}{B_{21}} \exp\left(\dfrac{h\nu}{KT}\right) - 1} \quad\quad (4.9.2)$$

- Equation (4.9.2) must agree with Planck's energy distribution formula which is given by,

$$Q = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{\exp\left(\dfrac{h\nu}{KT}\right) - 1} \quad\quad (4.9.3)$$

- Now by comparing Equations (4.9.2) and (4.9.3), we get,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad\quad (4.9.4)$$

$$\text{and} \quad\quad \frac{B_{12}}{B_{21}} = 1$$

$$\text{or} \quad\quad B_{12} = B_{21} \quad\quad \ldots(4.9.5)$$

Q. Compare light from ordinary source with laser light. (May 2015)

Ans. :

Difference between LASER and ordinary light

LASER	Ordinary light
It is highly monochromatic.	It is polychromatic.
It is highly coherent.	It is not coherent.
Stimulated emission is responsible for it.	Spontaneous emission is responsible for it.
Highly direction.	Not directional.
Highly energetic.	Poor energy is associated.
Example : He-Ne, Nd : YAG etc.	Example : Sunlight, LED

Syllabus Topic : Helium Neon Laser

Q.   With a neat energy level diagram describe the construction and working of He-Ne laser. What are its merits and demerits. (May 2013, Dec. 2014, May 2015)

Ans. :

Principle of He-Ne laser

- Gas lasers are usually employ a mixture of two gases say A and B where atoms of type A are initially excited by electron impact and they in turn transfer their energy to atoms of type B which are actual active centers. Here the energy transfer is done by atomic collisions between A and B where two of their energy levels are equal. It is also known as resonance transfer energy. It can be expressed as

$$e_1 + A = e_2 + A^*$$

$$A^* + B = A + B^*$$

- Here A^* and A represented the energy values of the atom of type A in metastable state and ground state respectively. B and B^* represent the energy values of the atom of type B in ground state and excited state respectively.

Construction

- The He-Ne laser consists of a long and narrow discharge tube which is filled with Helium and Neon in the ratio 10 : 1 with pressure of 1 mm of mercury. Flat glass quartz plates which function as Brewster windows are sealed to the tube at both ends. Two optically plane mirrors are fixed on either side of the tube normal to its axis as shown in Fig. 4.11.1.
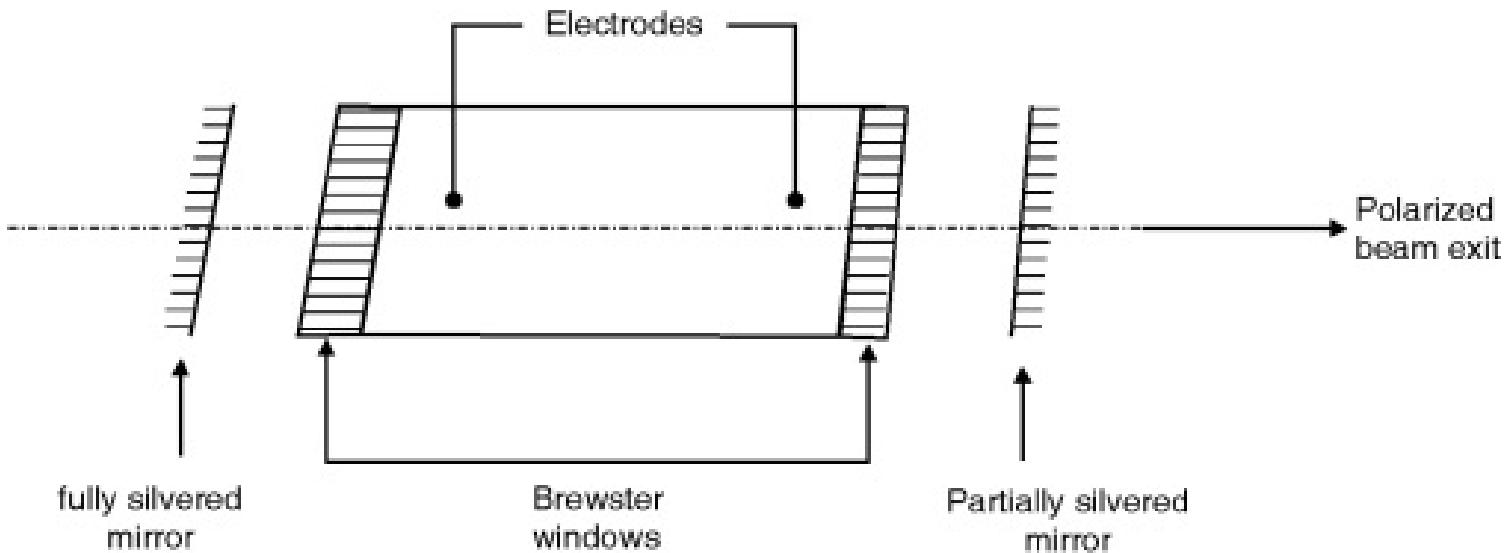
Fig. 4.11.1 : He-Ne laser tube (schematic presentation)

- One of the mirror is fully silvered with 100% reflectivity, where as the silvering of the other is slightly less so that 1% of the incident laser beam could be trapped by transmission.

Working

Energy level diagram :
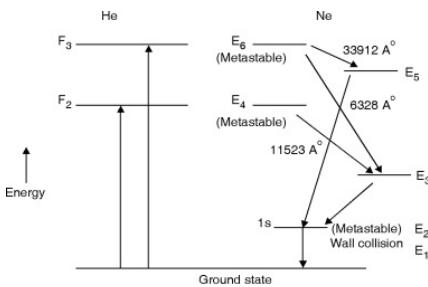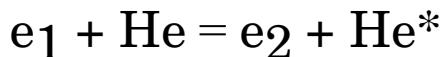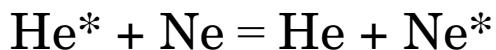
Fig. 4.11.2 : Energy level diagram of He-Ne

- When a voltage of about 1 kV is applied across the tube, a slow discharge of the gases is initialized in the tube. During the discharge, many electrons are rendered free from the gas atoms and are accelerated towards positive electrode and collide with Helium. Since He atoms are large in number, excitation of He takes place and as discussed in the principle above, we get

    $e_1 + He = e_2 + He^*$

    He atoms are excited to two energy levels F_2 and F_3 which are non-metastable states which leads to an increase of population in each of them.

- For Ne energy states metastable states E_6 and E_4 are very close to F_3 and F_2 of He atom*. Therefore when He atoms collides with a Neon atoms, because of the matching of energy levels, resonant transfer of energy takes place from He to Ne atoms. As a result, the Ne atoms get elevated to the E_6 and E_4 levels, whereas the He-atoms returns to ground state. This is represented as

    $He^* + Ne = He + Ne^*$

- The population increases rapidly and population inversion takes place between E_6 and E_4 with respect to E_5 and E_3.

- Three main transitions become available.

  (a) E_6 to E_5 gives rise to a radiation of wavelength 33912A° which is in infra-red region and hence not visible.

  (b) E_6 to E_3 gives rise to a radiation of wavelength 6328 A °, which is visible and of red colour.

  (c) E_4 to E_3 gives rise to a radiation of wavelength 11523 A° which is also in infra-red region.

- From E_5 and E_3 levels, atoms undergo spontaneous transitions to E_2 level at much faster rate. But E_2 level is metastable for Ne. The atoms will come down to ground state by wall collision. This effect is inversely proportional to the diameter of the discharge tube and hence the diameter of the He-Ne laser is only of few millimeters in diameter to enable efficient depopulation of E_2 level.

- Since the discharge in the tube is maintained continuously, the cycle of events also takes place continuously and the emission of laser is also continuous, because of which He-Ne laser is referred as a continuous wave laser.

Merits

(a) Continuous output.

(b) Highly monochromatic.

(c) Highly stable.

(d) No separate cooling is needed.

(e) As gases are found in pure form their optical properties are well defined.

Demerits

Very low output power.

Q.   With neat energy level diagram, explain principle, construction and working of Nd : YAG laser. (May 2016)

Ans. :

Principle of Nd : YAG laser

- It is a solid state laser.

- Nd represents Neodymium (Nd^{+3} ions are used).

- YAG represents Yittrium Aluminium Garnet ($Y_3 Al_5 O_{12}$).

- Some of the Y^{+3} ions are replaced by Nd^{+3}. The crystal atoms of YAG do not take part into lasing action, but serve as a host lattice in which Nd^{+3} resides.

Construction

- As shown in Fig. 4.12.1, an elliptically cylinder reflector with both of its axis occupied by a flash lamp and Nd : YAG rod respectively. The light leaving one focus of the ellipse will certainly pass through the other focus after reflection from reflecting surface. Hence entire light generated by flash tube is focussed on the Nd : YAG rod.

- Optical resonator is formed by highly silvered reflecting surfaces as shown in Fig. 4.12.1.
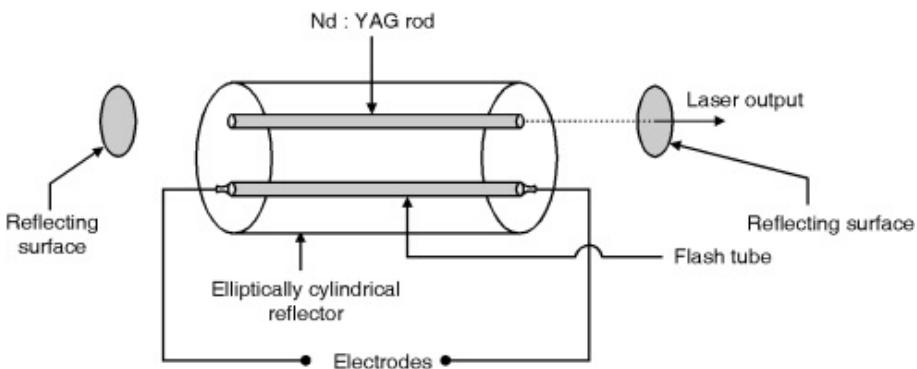
Fig. 4.12.1 : Nd : YAG laser

Working

- In Fig. 4.12.2 we have energy levels E_1, E_2 and E_3 of Nd along with many other levels of YAG. E_1 is ground state and E_3 offers metastable state.

- Pumping takes place with light of wavelength 5000 A° to 8000 A° which excites Nd^{+3} ions to higher states. The metastable state E_3 rapidly gets populated due to downward transitions from higher energy levels as none of them is metastable. Population inversion takes place between E_3 and E_2. A continuous laser of 10600 A° in infrared region is given out due to stimulate emission taking place between E_3 and E_2.
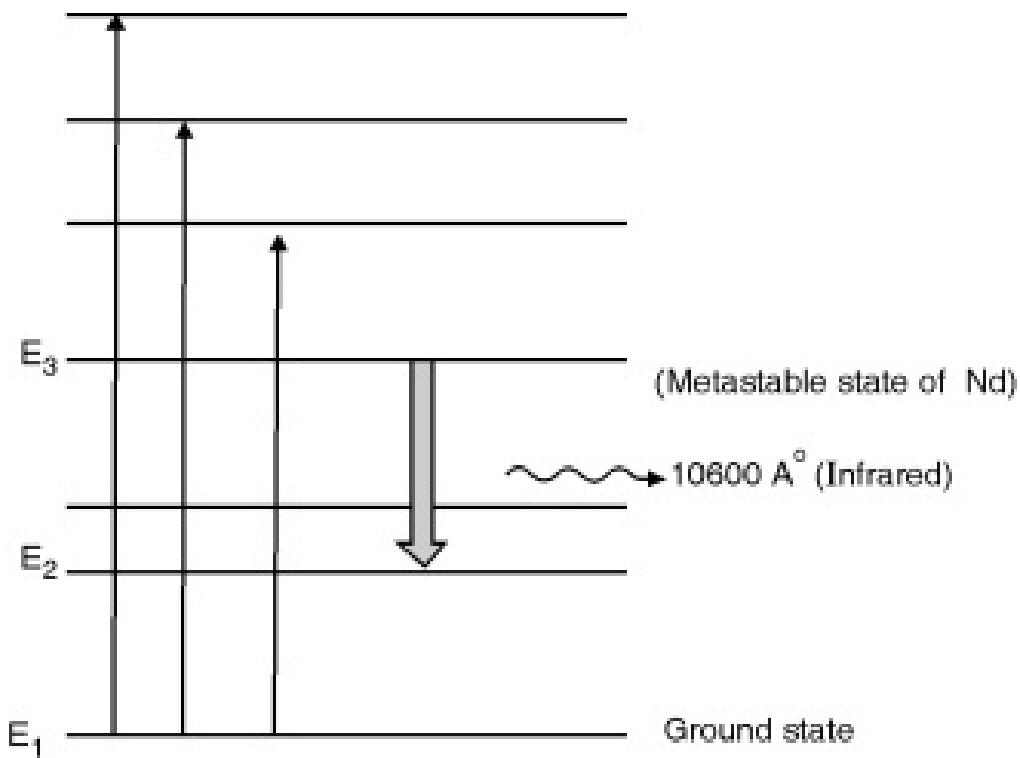
Fig. 4.12.2 : Energy levels of Nd along with YAG

Q. What are the merits and demerits of Semiconductor Diode Laser ?

Ans. :

Merits

(a) Simple and compact.

(b) Requires very little power and more efficient.

(c) Output can be controlled by controlling the junction current.

(d) Metastable state is not required.

Demerits

(a) Highly temperature sensitive.

(b) Less monochromatic

Syllabus Topic : Application of Lasers – Holography (Construction and Reconstruction of Holograms)

Q. What is holography ? Explain its construction and reconstruction with neat diagram.

(May 2014, Dec. 2015)

OR

Q. What is the fundamental principle of a Hologram ? How is it produced and how the real image is constructed by it. (Dec. 2016)

Ans. :

Holography

• The advent of lasers has made the art of holography

possible. Photography can be thought of as a new approach to the problem of generating images. An ordinary photograph represents a two dimensional recording of a three dimensional scene. The emulsion on the photographic plate is sensitive only to intensity variations. In this process the phase information carried by the electromagnetic wave scattered from the object is lost. Since only the intensity pattern is recorded, the 3D character of the object is lost.

- The principle behind the holograph is "During the recording process one superimposes on the scattered wave (emanating from the object) another coherent wave (called reference beam) of the same wavelength". These two waves interfere in the plane of the recording medium and produce interference fringes. This is known as recording process.

- The interference fringes-characteristic of the object is formed. The recording medium records the intensity distribution in the interference pattern. This interference pattern has recorded in it not only the amplitude distribution but also the phase of the electromagnetic wave scattered from the object. Since the recorded intensity pattern has both the amplitude and phase recorded in it has been called "hologram" (Holos in Greek means whole).

- The holograms have little resemblance to the object. It has in it a coded form of a wavefront. The reproduction of the image is known as reconstruction in which a wave identical to the one used as reference wave is used. When hologram is illuminated by the reconstruction wave, two waves are produced. One wave appears to diverge from the object and

provides the virtual image of the object. The second wave converges to form a second image which is real.
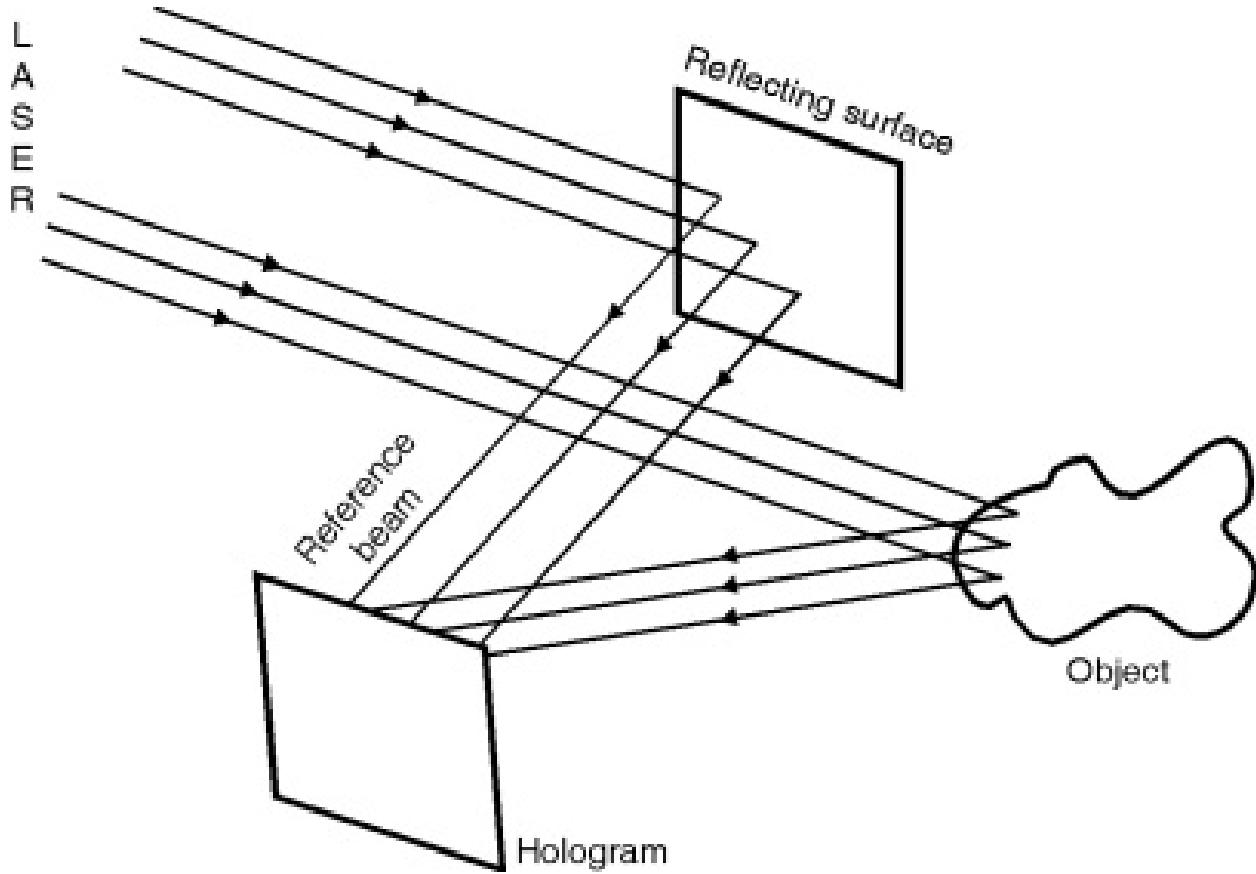
Fig. 4.14.1(a) : Construction of hologram
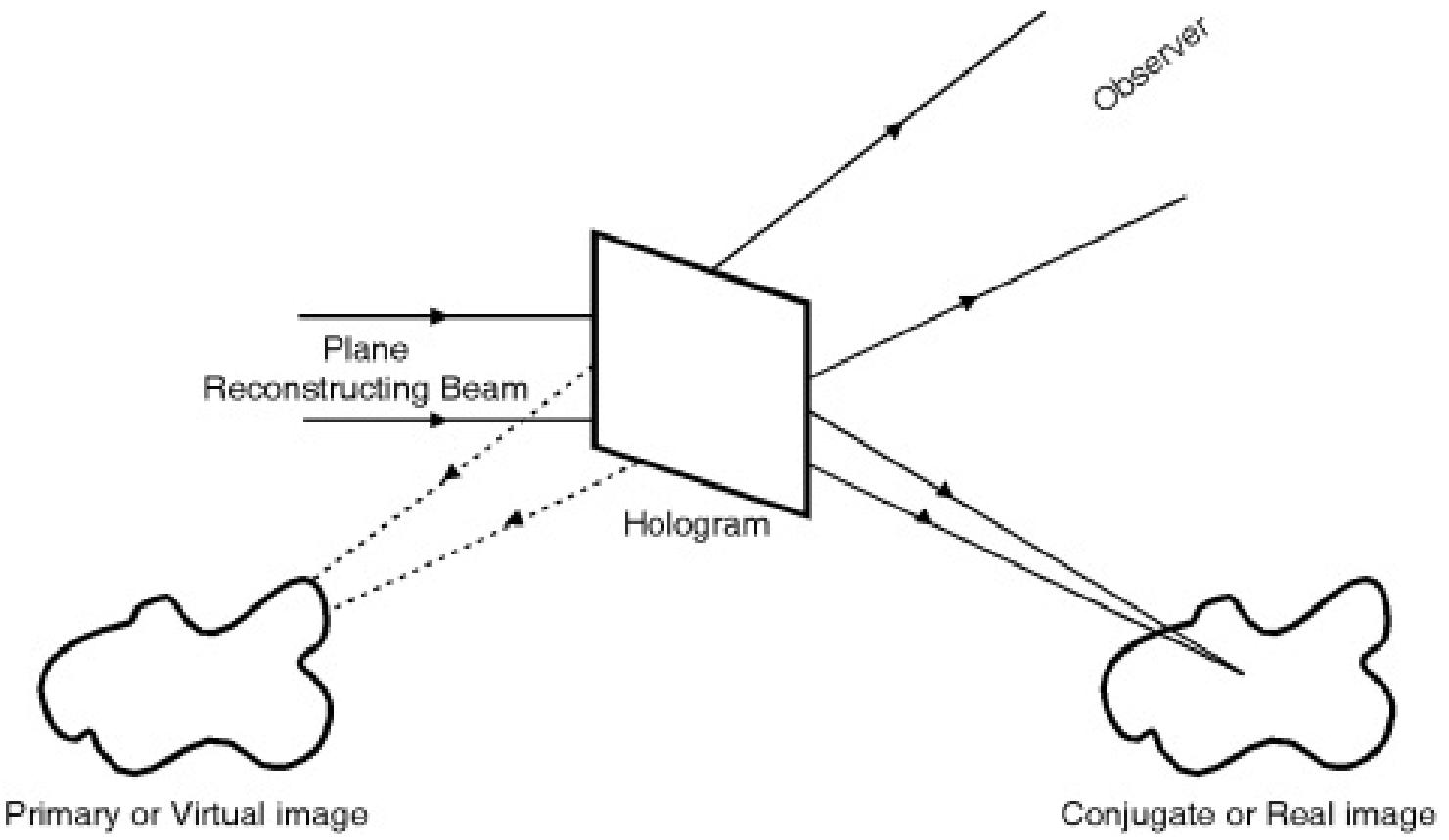
Fig. 4.14.1(b) : Reconstruction of image

Syllabus Topic : Industrial Application (Cutting, Welding)

Q. Explain Industrial and Medical Applications of Laser

Ans. :

Industrial and Medical Applications of Laser

Lasers widely used in the various industrial production purposes and material production. It is also used in the medical instruments. In industries, lasers are applied to a larger extent for the following processes.

(1) For welding and melting

- In very short time metal can be melt and then it will be evaporating. Thus accurate welding and melting of the hard material can be done very easily. Using laser

perfect and non porous joints of metals are possible. The typical laser welding machine is as shown in the Fig. 4.14.2.

- Due to increased power output it is possible to use this as a welding tool. Generally CO_2 gas laser is used as cutting tool. Thus advantages of laser welding are. Very high welding rate is possible.

- It is possible to weld dissimilar metals. Complex counters can be welded using easy turning of laser beam.

- Working material is not stressed because of non-contact method.

- The typical laser welding machine is as shown in Fig. 4.14.2.
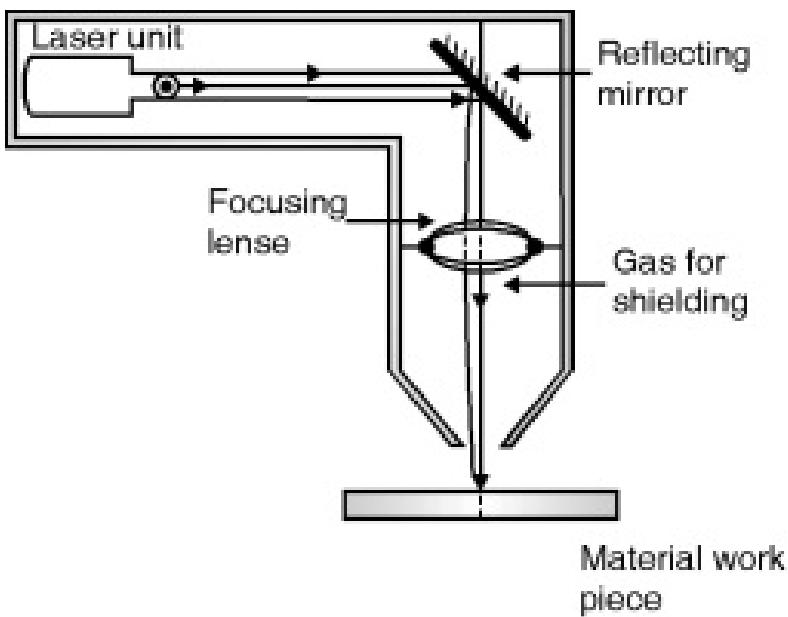
Fig. 4.14.2 : Laser welding machine

(2) For cutting and drilling holes

- For some engineering applications accurate cutting and holes are required it is difficult by normal cutting

and drilling machines. Generally high power pulse Nd: YAG laser is used for such operations. Pulsed laser is allowed to fall on the metal.

- The laser pulse increases the temperature of surface of metal. During the sort period its temperature exceeds above the boiling point of the metal, which results in accurate whole formation.

- The laser cutting of the materials is the common and useful application of the laser systems. During the cutting process the energy, absorbed by the material in the area in which the laser beam has immersed.

- The power densities about 10^5-10^8 W/cm^2 is transformed in to heat. This heat locally useful to a quick increase of the temperature of the metal piece; the fusion and/or the vaporization of the interaction zone determine the formation of accurate hole. If the passing hole has moved along the piece in working, it gives the separation of the two cutting pieces.

Also LASER finds its applications in

1) Cutting, drilling of hard metals and diamonds.

2) In surgery, these are used because due to the production of localized heat, they cut and seal all blood vessels simultaneously. This is known as bloodless surgery. To repair detached retina, lasers are very useful.

3) Automatic control of rockets and satellites.

4) Used in warfare of detecting and destroying missiles, tanks etc.

5) High speed photography.

Solved Problems

Ex. 4.15.1

Find the ratio of population of the two energy states of the active medium producing laser transition between which has wavelength 694.3 nm. Assume temperature 27°C.

Soln. :

Data : $\lambda = 694.3 \times 10^{-9}$ m, T = 300 °K

To find : $\dfrac{N}{N_0}$

Let ΔE be the energy difference between the two energy states

$$\therefore \quad N = N_0 \, e^{-\Delta E/KT}$$

$$\therefore \quad \frac{N}{N_0} = e^{-\Delta E/KT}$$

$$\text{Now} \quad \Delta E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-24} \times 3 \times 10^{5}}{694.3 \times 10^{-9}}$$

$$= 2.865 \times 10^{-19}$$

$$\therefore \quad \frac{N}{N_0} = e^{-\frac{2.865 \times 10^{-19}}{138 \times 10^{-23} \times 300}}$$

$$= e^{-69.2}$$

$$\frac{N}{N_0} = 8.82 \times 10^{-31} \qquad \text{...Ans.}$$

Ex. 4.15.2

A He-Ne laser is emitting a laser beam with an average power of 4.5 mW. Find the number of photons emitted per second by the laser. The wavelength emitted is 6328 A°.

Soln. :

Data :   Wavelength $\lambda = 6328 \times 10^{-10}$ m, Power = 4.5 mW = 4.5 $\times 10^{-3}$ W

To find : Number of photons emitted/second

We know that energy difference

$$\Delta E = h\nu = \frac{hc}{\lambda} = \frac{6.634 \times 10^{-34} \times 3 \times 10^{8}}{6328 \times 10^{-10}}$$

$$= 3.143 \times 10^{-19} \text{ J}$$

This energy difference becomes the energy of each of the emitted photon. If N is the number of photons emitted per second to give a power output of 4.5 mW then,

$$N \times \Delta E = 4.5 \times 10^{-3} \text{ W} = 4.5 \times 10^{-3} \frac{J}{sec}$$

$$= \frac{4.5 \times 10^{-3}}{3.143 \times 10^{-19}} = 1.43 \times 10^{16}$$

∴ Number of photons emitted per second is 1.43×10^{16}

...Ans.

Ex. 4.15.3

A pulsed laser emits photons of wavelength 780 nm with 20 mW average power/pulse. Calculate the number of photons contained in each pulse if pulse duration is 10 ns.

Soln. :

Data :   Wavelength of the photon $\lambda = 780 \times 10^{-9}$ m

Power of each pulse P = 20×10^{-3} J/sec.

Duration of each pulse = t = 10 ns = 10×10^{-9} s

To find : Number of photon in each pulse N = ?

$$\text{Energy of each photon} = \Delta E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{780 \times 10^{-9}}$$

$$= 2.55 \times 10^{-19} \text{ J}$$

Energy of each pulse = power × time = $20 \times 10^{-3} \times 10 \times 10^{-9}$

$$= 2 \times 10^{-10} \text{ J}$$

Now $\quad N \times \Delta E = E$

$$\therefore N = \frac{E}{\Delta E} = \frac{2 \times 10^{-10}}{2.546 \times 10^{-19}} = 7.86 \times 10^8$$

$\therefore$ Number of photons in each pulse is 7.86×10^8

...Ans.

Ex. 4.15.4

A laser beam of intensity $7.68 \times 106 \text{ mW/m}^2$. It has an aperture of 5×10^{-3} m and it emits light of wavelength 7000 A°. The beam is focused with a lens of focal length 0.1 m, calculate the area and power of the image.

Soln. :

Given : $\lambda = 7200 \times 10^{-8}$ m, $\quad$ f = 0.1 m, $\quad$ a = 5×10^{-3} m , $\quad$ I = $7.68 \times 10^6 \text{ mW/m}^2$.

Formula : Hint : for a wavelength λ of laser light, let a be the radius of beam and f is the focal length of the lens, then the incoming beam get focused into a region of area of radius r is given by,

$r = \frac{\lambda f}{a} \quad P = I \pi r^2$

where , 'λ' is wavelength of laser beam light, 'a' is the radius of beam.

f is the focal length of the lens and r is the region of radius in which the incoming beam focused.

Substituting the values we get,

$r = \frac{7200 \times 10^{-8} \times 0.1}{5 \times 10^{-3}}$

$r = 1.44 \times 10^{-3}$ m

Intensity I of beam is = $7.68 \times 10^6 \text{ mW/m}^2$.

$$P = 7.68 \times 10^6 \, 3.14 \, (1.44 \times 10^{-3})^2$$

$$I = 50.0 \text{ mW}$$

$$P = 50 \text{ mW}$$

Power of intense laser beam is beam is P = 50 mW.   ...

Ans.

Ex. 4.15.5

Energy transitions in the laser have two states at different temperature one is at 300 °K and another at 500 °K. If it emits light of wavelength 7000 A°, then determine the relative population.

Soln. :

Given : $T_1 = 350$ K, $T_2 = 550$ K, $\lambda = 7000$

Formula : $E_2 - E_1 = \Delta E = h\nu = hc/\lambda$, Population ratio is $\dfrac{N_2}{N_1} = e^{-(E_2 - E_1)/KT}$

Substituting the given values with standard physical data we get,

$$\Delta E = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{7000 \times 10^{-10}}$$

$$\Delta E = 2.839 \times 10^{-19} \text{ J}$$

$$\Delta E = 1.772 \text{ eV}$$

Population at 350 °K,

$$\frac{N_2}{N_1} = \exp\left[\frac{-1.772}{8.6 \times 10^{-5} \times 350° \text{ K}}\right]$$

$$= e^{-58}$$

$$\frac{N_2}{N_1} = 6.4 \times 10^{-26}$$

Population at, 550 °K

$$\frac{N_2}{N_1} = \exp\left[\frac{-1.77}{8.6 \times 10^{-5} \times 550^\circ \text{ K}}\right] = e^{-37.52}$$

$$= 5.07 \times 10^{-7}$$

Population at 350 °K, is 6.4×10^{-26} and at 550 °K is 5.07×10^{-7} …Ans.

Ex. 4.15.6

A laser emits radiation of wavelength 8000 A°, in this phenomenon 2.8×10^{19} total number of ions (photons) are contributed. Calculate total energy per laser pulse, if energy of emitted photon is 1.55 eV.

Soln. :

Given : $\lambda = 8000 \times 10^{-10}$ m

Total photons $= 2.8 \times 10^{19}$

$E = 1.55$ eV $= 1.55 \times 1.6 \times 10^{-19}$ J.

Formula :

Energy per pulse = Energy of single photon / ion × Total number of ions

Substituting the values we get,

Energy / pulse $= 1.55 \times 1.6 \times 10^{-19} \times 2.8 \times 10^{19}$

$= 1.55 \times 1.61 \times 2.8 = 6.98$ J

Energy per pulse = 6.98 Joule

…Ans.

➜➜➜

Fibre Optics

Q.   Define : Total internal reflection. (Dec. 2014, Dec. 2015)

Ans. :

Total internal reflection

- If light wave enters at one end of a fibre, most of it propagates down the length of the fibre and comes out from the other end of the fibre.

- There may be some loss due to leakages through side walls of the fibre. One should know how light gets guided through the fibre. The reason for confining the light beam inside the fibre is "Total Internal Reflection" (TIR). Fibre obeys laws of "reflection and refraction of light waves".

Q.   What is monomode and multimode fibre ? (Dec. 2016)

Ans. :

Monomode fibre

Single mode fibre (SMF) has a smaller core diameter and

can support only one mode of propagation.

Multimode fibre

Multi mode fibre (MMF) has larger core diameter and supports a number of modes.

Q.   Differentiate between S.I. fibre and GRIN fibre. (May 2014)

Ans. :

Difference between SI and GRIN Fibre

Sr. No.	SI	GRIN
1.	Discontinuity of index profile at core-cladding junction.	R.I of core decreases gradually to attain R.I of cladding at core-cladding junction.
2.	R.I. of core is constant.	R.I. of core decreases nearly in parabolic manner.
3.	High attenuation	Low attenuation
4.	For a given diameter the N.A is greater	For a given diameter the N.A is lesser compare to SI

Syllabus Topic : Numerical Aperture, Angle of Acceptance

Q.   What is NA ? (Dec. 2013, Dec. 2014, Dec. 2015)

Ans. :

Numerical Aperture (NA)

The numerical aperture is defined as the sine of the acceptance angle.

$$NA = \sqrt{n_1^2 - n_2^2}$$

Numerical aperture determines the light gathering ability of the fibre. It is a measure of the amount of light that can be accepted by the fibre.

Q.   Derive the expression for N.A. for step Index fibre. (May 2014, May 2015)

Ans. :

Numerical Aperture (NA) of step Index fibre

- In SI fibre any light wave which travels along the core and meets the cladding, with $\theta > \theta_c$, will be totally reflected. This reflected ray will then meet the opposite surface of the cladding again at the $\theta > \theta_c$ and so it will be again totally reflected. Any other light wave, which is meeting the core cladding interface at or above θ_c, will also be totally reflected and hence will propagate along the core.

- However, any light wave meeting the core cladding interface at any angle below θ_c will pass into and be absorbed by cladding. For a fibre for which $\mu_1 > \mu_2$, light impinging on the core within a critical angle θ_c, is coupled into the fibre and will propagate.

- If the external incident angle is θ_0 corresponding to the critical angle of incidence θ_c at the core-cladding interface of the fibre as shown in Fig. 5.6.1, the light will stay in the fibre.

- So any light wave impinging on the core within this maximum external incident angle θ_0 is coupled into the fibre and will propagate. This angle is called as angle of acceptance.

Fig. 5.6.1

- Let us now derive a mathematical relation for the acceptance angle θ_0.
  Refer Figs. 5.6.2(a) and (b).

(a)   (b)

Fig. 5.6.2

- Now from Figs. 5.6.2(a) and (b) and applying law of refraction at A and B we get,

$$\mu_0 \sin \theta_0 = \mu_1 \sin (90 - \theta_c) \qquad \ldots(5.6.1)$$

$$\mu_1 \sin \theta_c = \mu_2 \sin 90 \qquad \ldots(5.6.2)$$

$\sin \theta_c = \frac{\mu_2}{\mu_1}$ and $\mu_0 \sin \theta_0 = \mu_1 \cos \theta_c$

$$\mu_0 \sin \theta_0 = \mu_1 \sqrt{1 - \sin^2 \theta_c}$$

$$\mu_0 \sin \theta_0 = \mu_1 \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}} = \sqrt{\mu_1^2 - \mu_2^2}$$

$$\therefore \sin \theta_0 = \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0} \qquad \ldots(5.6.3)$$

$$\text{Acceptance angle } \theta_0 = \sin^{-1} \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0} \qquad \ldots(5.6.4)$$

- If fibre has surrounding medium as air, $\mu_0 = 1$

$$\therefore \text{Acceptance angle} \qquad \theta_0 = \sin^{-1} \sqrt{\mu_1^2 - \mu_2^2} \qquad \ldots(5.6.5)$$

Q.   Define the terms : Acceptance angle. (Dec. 2014, Dec. 2015)

Ans. :

Acceptance angle

Any light wave impinging on the core within this maximum external incident angle θ_0 is coupled into the fibre and will propagate. This angle is called as angle of acceptance.

Ex. 5.6.1 (Dec. 2014, 3 Marks)

The N.A. of an optical fibre is 0.5 and core R.I. is 1.54. Find R.I.

of cladding.

Soln. :

$$N.A. = 0.5, \quad n_1 = 1.54$$

To find n_2

Formula $\quad N.A = \left(\sqrt{n_1^2 - n_2^2} \right)$

$$4 n_2 = \left(\sqrt{n_1^2 - (N.A)^2} \right) = \sqrt{1.54^2 - 0.5^2} = 1.4565 \quad \text{... Ans.}$$

Syllabus Topic : V Number

Q.   Explain the term V-number. (Dec. 2016)

Ans. :

V-Number

- In optical fibre the light propagates in the same way as electromagnetic wave propagates. When confined to a duct or guide, it propagates like electromagnetic wave, but at a much higher frequency. In optical fibre communication, two bands are presently famous for their low losses.

    (1)   800 nm to 900 nm

    (2)   1200 nm to 1400 nm

- The propagation is possible through number of modes like TE, TM, hybrid HE and hybrid EH (Discussion beyond the scope of this syllabus). It is also important to know that all modes will not be there in a single fibre. The number of modes supported by a fibre is determined by an important parameter called "cut off parameter". It is also termed as "normalized frequency " or "cut off" or "V-number".

- The mathematical expression for the V-number is,

$$V = \pi \frac{d}{\lambda_0} \sqrt{\mu_1^2 - \mu_2^2} \qquad \ldots(5.8.1)$$

Where, λ_0 = free space wavelength

d = 2a = diameter of the core (a = radius of the core)

$$\therefore V = \frac{\pi d}{\lambda} (NA)$$

$$= \frac{\pi d}{\lambda} \mu_1 \sqrt{2\Delta} \qquad \ldots(5.8.2)$$

- The maximum number of modes N_m supported by SI fibre is given by

$$N_n \simeq \frac{1}{2} V^2 \quad \text{(provided V-number is considerably larger than unity)}$$

Syllabus Topic : Losses in Optical Fibre (Attenuation and Dispersion)

Q.   What is dispersion in optical fibers' ? Mention any three dispersion you have studied and explain one in detail. (May 2013)

Ans. :

Dispersion

The light pulses that propagate through a fibre suffer several dispersion effects. The dispersion effects spread the output pulse and changes its shape, so that it may merge into the succeeding or previous pulses as shown in Fig. 5.9.1. Hence pulses are separated out, affecting the maximum bit rate.

Types of dispersions

(a)   Intermodal dispersion

(b)   Waveguide dispersion

(c)   Material dispersion
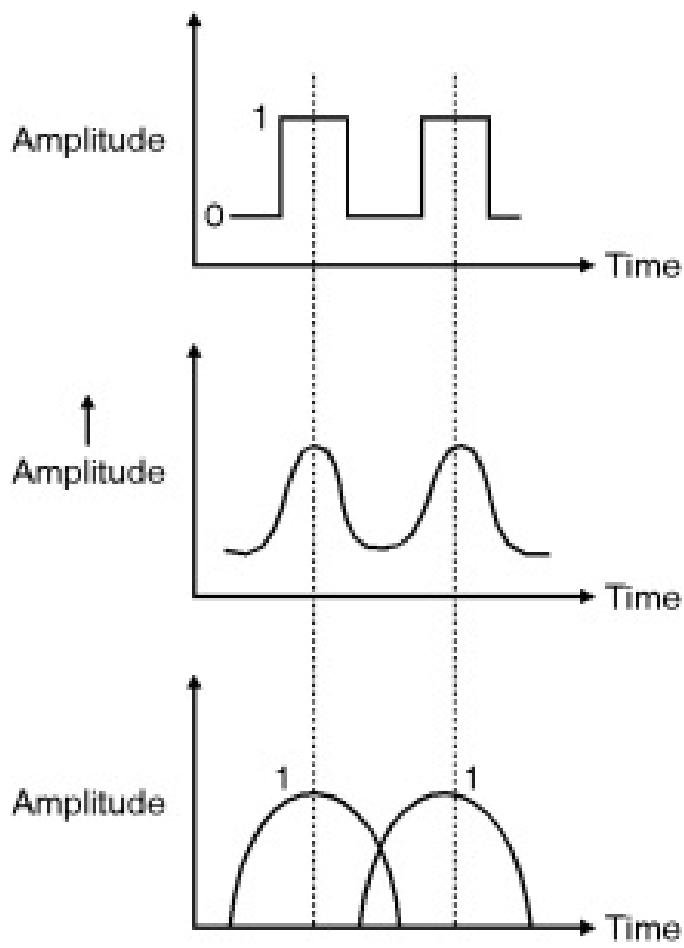
Fig. 5.9.1 : Variation in waveform

Intermodal dispersion

The rays following zig-zag paths while traveling in an optical fibre, and hence the rays, under different modes will travel different total lengths of path and emerge at the far end at slightly different times. This is known as intermodal dispersion.

Q.   What are the advantages of using an optical fiber ? (May 2015)

OR

Q. Why would you recommend use of optical fibre in communication system ?

Ans. :

Advantages of optical fibre

(i) Through optical fibre light rays passes using concept of Total Internal Reflection (TIR) and hence there is no loss up to few kilometers.

(ii) Diameter of the fibre is very small and hence network made of optical fibre requires very less space.

(iii) The power required to operate it is very small compare to conventional copper wire.

(iv) Since optical fibres are made of either silica or plastic and hence it is very cheap compare to its metallic equivalent.

(v) Optical fibres are not much affected by parameters like pressure, temperature, twist, salinity etc. (Except for specially designed fibres)

Use of optical fibre in Communication system

- The fibre optical commutations system is very much similar to that of a traditional communication system. Fig. 5.10.1 shows optical fibre communication system Light waves which are carrier waves, are generated by a LED or a laser diode.

- The intensity of light is proportional to the current passing through it. When the message in electrical form is fed to the optic source, the light output from it follows the variations of message. In order to have efficient transmission through the fibre the frequency of operation is

chosen to lie in the optical window region of the fibre.

- The optical signal traveling through the fibre will become progressively attenuated and distorted because of various losses and dispersion in the fibre. Hence repeaters are used in the transmission line to amplify and reshape the signal.
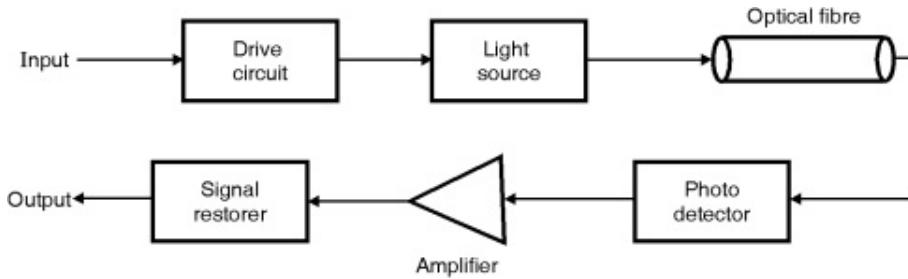
Fig. 5.10.1

- At the received end, an output coupler directs the light emerging from the fibre channel to a photo detector. The photo detector converts the light wave into an electric current. The message is contained in the detector output current.

- The detector output current is filtered to extract the message and it is amplified if needed.

- It is then fed to a suitable transducer to convert into an audio or video form.

Solved Problems

Ex. 5.11.1 (May 2013, 3 Marks)

Calculate the numerical aperture of a fibre with core index n_1 = 1.55 and cladding index, n_2 = 1.51.

Soln. :

Given : $n_1 = 1.55$, $n_2 = 1.51$

To find : NA = ?

Formula :

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.55^2 - 1.51^2} = 0.35 \quad \text{...Ans.}$$

Ex. 5.11.2 (May 2014, 3 Marks)

An optical glass fibre of R.I. 1.50 is to be clad with another glass to ensure internal reflection that will contain light travelling within 5° of the fibre axis. What minimum index of refraction is allowed for the cladding?

Soln. :

$$\theta_{max} = 5°, \quad n_1 = 1.5$$

To find : n_2

Formula :
$$\theta_m = \sin^{-1}(N.A) = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$$

$$4 \sin \theta_m = \left(\sqrt{n_1^2 - n_2^2}\right)$$

$$n_2 = \left(\sqrt{n_1^2 - \sin^2 \theta_m}\right) = 1.4974 \quad \text{... Ans.}$$

Ex. 5.11.4 (Dec. 2016, 3 Marks)

A fibre cable has an acceptance angle of 30° and core index of R.I. 1.4. Calculate R.I. of cladding.

Soln. :

$$\theta = 30°, \quad n_1 = 1.4$$

To find : $n_2 = ?$

$$\text{Acceptance angle } \theta = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$$

$$\sin 30° = \sqrt{n_1^2 - n_2^2}$$
$$0.5 = 1.4^2 - n_2^2$$

$$\therefore \quad n_2 = 1.308 \quad \text{...Ans.}$$

Ex. 5.11.10 (Dec. 2015, 3 Marks)

The core diameter of multimode step Index fibre is 50 μm. The numerical aperture is 0.25. Calculate the no. of guided modes at an operating wavelength of 0.75 μm.

Soln. :

Given : $d = 50$ μm

                $NA = 0.25$

                $\lambda_0 = 0.75$ μm

To find : $N = $ Number of modes

$N = \dfrac{V^2}{2}$

Here $V = \dfrac{\pi d}{\lambda_0} NA = 52.36$

∴ Number of modes $= \dfrac{52.36^2}{2} = 1371$ ...Ans.

Ex. 5.11.11 (May 2013, 3 Marks)

Compute the maximum radius allowed for a fibre having core refractive index 1.47 and cladding refractive index 1.46. The fibre is to support only one mode at a wavelength of 1300 nm.

Soln. :

        The condition for single mode is $V < 2.405$.

or $\dfrac{2\pi a}{\lambda}\sqrt{\mu_1^2 - \mu_2^2} < 2.405$

or $a < \dfrac{2.405}{2\pi \sqrt{\mu_1^2 - \mu_2^2}} \lambda$

or $a < \dfrac{5.82}{2}$ μm

The   maximum   radius $=$ 2.91   μm
                ...Ans.

Ex. 5.11.12 (May 2015, 3 Marks)

Calculate V number for an optical fiber having numerical aperture 0.25 and core diameter 20 μm if it is operated at 1.55 μm.

Soln. :

        $NA = 0.25$

Core diameter $= 20 \ \mu m$

$\lambda_0 = 1.55 \ \mu m$

Formula

$$V = \frac{\pi d}{\lambda_0} \times (NA) = \frac{\pi \times 20 \times 10^{-6}}{1.55 \times 10^{-6}} \times 0.25$$

$V = 10.13$ ...Ans.

Ex. 5.11.13 (Dec. 2013, 3 Marks)

Compute the cut-off parameter and the number of modes supported by a fibre μ_1 (core) $= 1.53$ and μ_2 (cladding) $= 1.5$; core radius 50 μm and operating wavelength is 1 μm.

Soln. :

$$V = \frac{2 \pi a}{\lambda}. NA \ (\text{cut off parameter})$$

$$NA = \sqrt{\mu_1^2 - \mu_2^2} = 0.3015$$

$V = 94.72$

$$\text{The number of modes} = \frac{V^2}{2} = 4485.85 \qquad \text{...Ans.}$$

Ex. 5.11.14 (May 2016, 7 Marks)

An optical fibre has core diameter of 6 μm and its core R.I. is 1.45. The critical angle is 87°. Calculate (i) R.I. of cladding (ii) Acceptance angle (iii) The number of modes propagating through fibre when wavelength of light is 1 μm.

Soln. :

Given : $n_1 = 45$, $d = 6 \ \mu m$

Critical acceptance angle $\theta_c = 87°$

∴ R.I. of cladding,

$$\text{as} \quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\therefore \quad \sin(87) = \frac{n_2}{n_1}$$

$$\therefore \quad n_2 = n_1 \sin(87°) = 1.45 \times \sin 87°$$

$$= 1.448$$

$$\therefore \text{R.I. of cladding} = 1.448 \qquad \text{...Ans.}$$

Now acceptance angle θ_0

$$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

$$= \sin^{-1}\left(\sqrt{1.45^2 - 1.448^2}\right)$$

$$= 4.366° \qquad \text{...Ans.}$$

As number of modes propagating through fibre is $N = \dfrac{V^2}{2}$

$$\text{Here } V = \frac{\pi d}{\lambda_0}\left(\sqrt{n_1^2 - n_2^2}\right)$$

$$\therefore N = \frac{1}{2}\left[\frac{\pi^2 d^2}{\lambda_0^2} \times (n_1^2 - n_2^2)\right]$$

$$= \frac{1}{2}\left[\pi^2\left[\frac{6}{1}\right]^2 \times (1.45^2 - 1.448^2)\right]$$

$$= 1.029 \qquad \text{...Ans.}$$

Ex. 5.11.15 (Dec. 2016, 7 Marks)

Calculate the number of modes for an optical fibre of core diameter 40 μm will transmit as its core and cladding R.I. are 1.5 and 1.46 respectively. Wavelength of light used is 1.5 μm.

Soln. :

Given : $n_1 = 1.5$, $n_2 = 1.46$, $\lambda_0 = 1.5$ μm, $d = 40$ μm

$$\text{as} \quad N = \frac{V^2}{2}$$

$$\text{and} \quad V = \frac{\pi d}{\lambda_0}\left(\sqrt{n_1^2 - n_2^2}\right)$$

$$\therefore N = \frac{1}{2}\left[\frac{\pi d}{\lambda_0}\left(\sqrt{n_1^2 - n_2^2}\right)\right]^2$$

$$= \frac{1}{2}\left[\pi \times \left(\frac{40}{1.5} \right) \times \sqrt{(1.5)^2 - (1.46)^2} \right]^2$$

$$= 415.48 \qquad \qquad \text{...Ans.}$$

→ → →

Q.   Explain Cartesian Coordinate System.

Ans. :

Cartesian Coordinate System

(i)   Point in Cartesian system
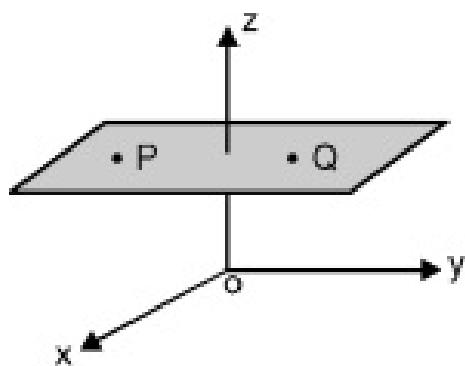
Fig. 6.1.1 : Z = Constant plane   Fig. 6.1.2 : Point is defined by three planes
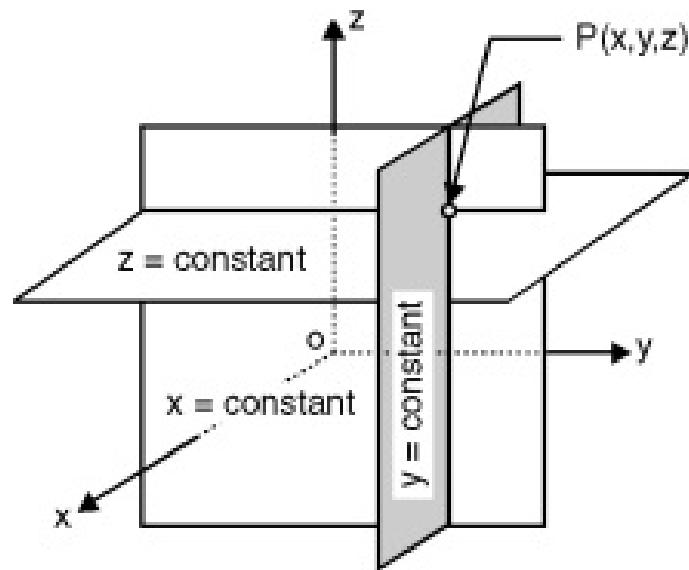
- Consider a plane perpendicular to the z-axis as show in Fig. 6.1.1. As, z-axis is vertical, a plane normal to z is horizontal, also xy plane is horizontal. Therefore xy plane and plane under consideration are parallel.

- The perpendicular distance of any point on this plane from xy plane will be the same. The distance from xy plane is nothing but z coordinate. Thus any point on the plane perpendicular to z-axis is having the same z coordinate. In other words we can say that for a plane perpendicular to z-axis, z coordinate of any point on the plane is constant or simply the plane is defined as z = constant plane.

- Similarly, a plane perpendicular to x-axis can be defined as x = constant plane and a plane perpendicular to y-axis is y = constant plane. Remember, x = constant and y = constant planes are vertical planes, while z = constant plane is horizontal plane.

- Consider, x = constant and y = constant planes. These two vertical planes intersect and the intersection is a vertical line. To obtain a point take the third plane, i.e. z = constant plane i.e. horizontal plane. The vertical line and horizontal plane intersects and the intersection is a point [point P in Fig. 6.1.2].

- Thus, point P is a intersection of x = constant, y = constant and z = constant planes and coordinates of it can be written as (x, y, z).

Any point in Cartesian system is the intersection of x = constant, y = constant and z = constant planes.

Point in Cartesian system : (x, y, z)

- These planes are mutually perpendicular to each other. This is a very important concept. So, if the coordinates of a point are (2, –1, 3) it means, it is the intersection of x = 2, y = –1 and z = 3 planes.

(ii) Unit Vectors in Cartesian System
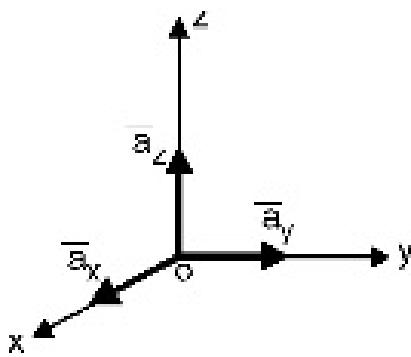
Fig. 6.1.3 : Unit vectors
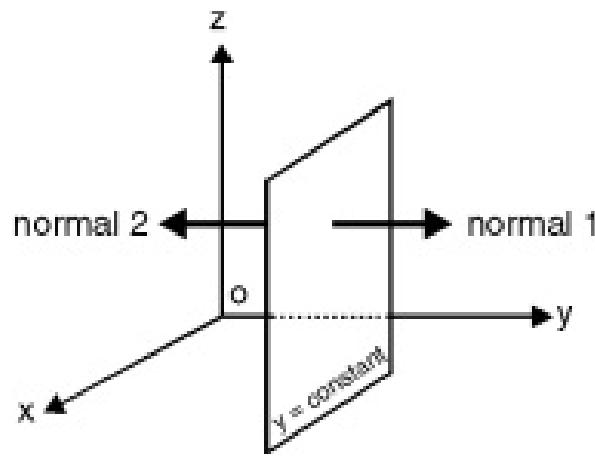
Fig. 6.1.4 : Unit vector $\bar{a}_y$

- Unit vectors in Cartesian system are $\bar{a}_x$, $\bar{a}_y$ and $\bar{a}_z$. These vectors are in the positive x, y and z directions as shown in Fig. 6.1.3. Instead of $\bar{a}_x$, $\bar{a}_y$ and $\bar{a}_z$ many times different notations are used like or $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$.

- We are using first notations for unit vectors in Cartesian system throughout this book. There is one more way to describe directions of unit vectors.

> **Unit vectors in Cartesian system : $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$**

Syllabus Topic : Cylindrical Coordinate System

Q. Derive necessary expressions to convert cylindrical co-ordinates to Cartesian co-ordinates.

Ans. :

Conversion of Cylindrical coordinates to Cartesian coordinates

- In the beginning of this article, we have seen that cylindrical system is built by taking x, y, z axes for reference. When we write a point as P (r, ϕ, z) it must have some Cartesian coordinates associated with it. Here we are

going to find relation between r, ϕ, z in cylindrical and x, y, z in Cartesian.

- To find x and y corresponding to P (r, ϕ, z), project pt. P on xy plane i.e. point Q. Project OQ on x and y axes. The length of projection on x-axis gives x coordinate (length OR) and length of projection on y axis (OS) gives y coordinate. In Fig. 6.1.7.
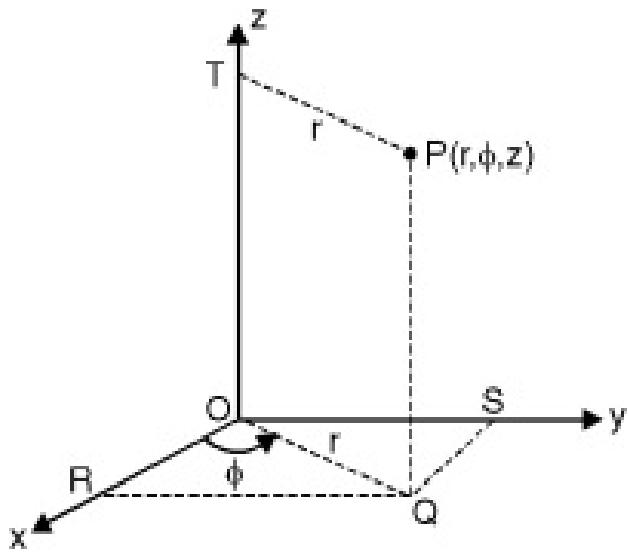
Fig. 6.1.7 : To show co-ordinate transformation

$OR = OQ \cos \phi$

i.e. $x = r \cos \phi$ ... (6.1.1)

and $OS = r \cos (90 - \phi)$

i.e. $y = r \sin \phi$ ... (6.1.2)

And z in Cartesian is same as z in cylindrical

∴ $z = z$ ...(6.1.3)

Syllabus Topic : Spherical Coordinate System

Q. Explain spherical co-ordinate system and hence explain its conversion to cartesian system.

Ans. :

Spherical Coordinate System

- For spherical coordinate system also x, y, z axes are used for reference. Imagine a sphere of radius r with center at origin. Any point on the sphere is at the same distance r from origin, therefore the spherical surface is defined as r = constant surface.

- Now consider a line from origin making angle θ with z-axis. Rotate this line about z-axis fixing the end at the origin. This forms a cone with angle θ, this conical surface is defined as a θ = constant surface.

- When a sphere with center at origin intersects with a vertical cone with vertex at origin, the intersection is a horizontal circle with radius equal to r sin θ [see Fig. 6.1.8].

- We want to locate a point in spherical coordinate system. Imagine a ϕ = constant plane similar to in cylindrical system. A horizontal circle with center on z-axis, intersects ϕ = constant plane. The intersection is a point.

> **Point in spherical system : (r, θ, ϕ)**

- Because r = constant, θ = constant and ϕ = constant surface intersects at a point, the point is defined as (r, θ ϕ). In spherical system variation of angle θ is from 0 to 180° and variation of ϕ is from 0 to 360°.
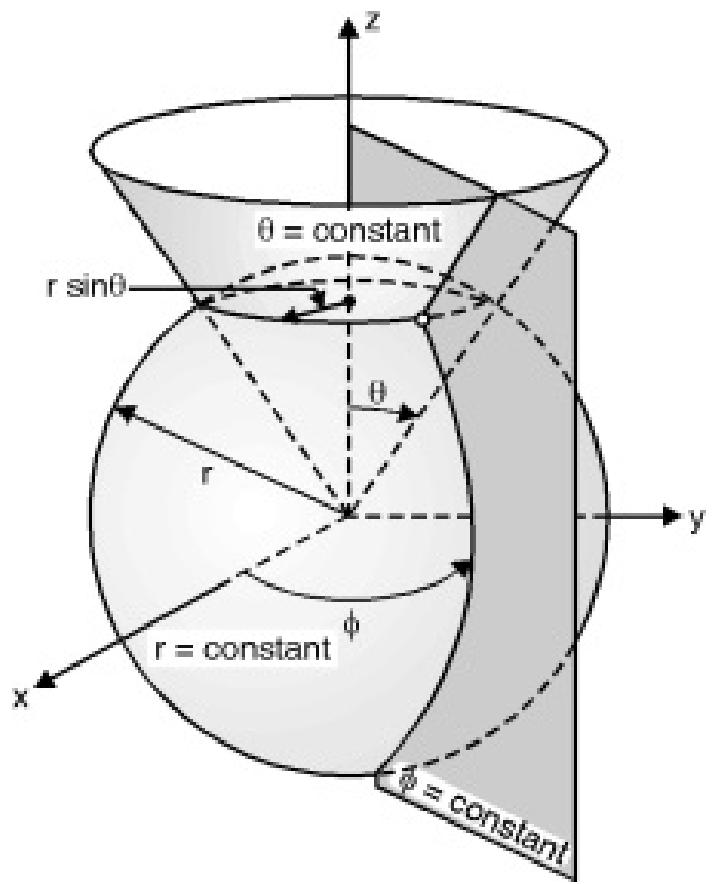
Fig. 6.1.8 : Spherical Coordinates
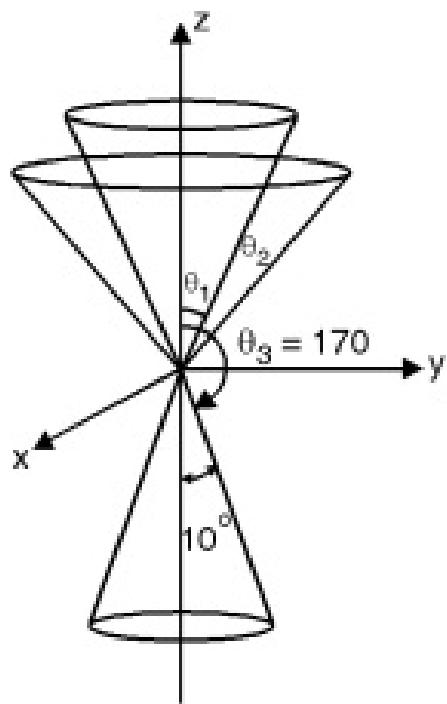
Fig. 6.1.9 : Variation of θ from 0 to 180° only

Conversion of Cartesian from Spherical Coordinates

- The unit vectors in spherical system are $\bar{a}_r, \bar{a}_\theta$ and $\bar{a}_\phi$. These unit vectors are perpendicular to r = constant, θ = constant, and φ = constant surface respectively and in the increasing directions of the r, θ and φ respectively. In Fig. 6.1.10 only the first octant is shown for simplicity, and the unit vectors are also shown in Fig. 6.1.10.
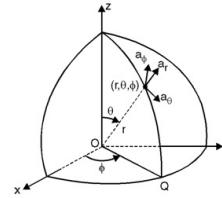
Fig. 6.1.10 : Unit vectors in spherical System
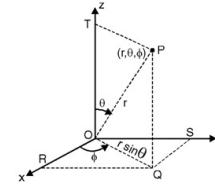
Fig. 6.1.11 : Relation between Spherical and Cartesian coordinates

- When point P (r, θ φ) is present in Cartesian system, it must have corresponding Cartesian coordinates. Here we are going to find relation between spherical coordinates and Cartesian coordinates.

- To find x and y corresponding to point P, project point P in xy plane (Fig. 6.1.11). The projection is OQ = r sin θ. Now its projection on x and y axis gives x and y coordinates corresponding to point P.

Projection of OQ on x-axis is

$$OR = OQ \cos \phi = r \sin \theta \cos \phi$$

$$\therefore \quad x = r \sin \theta \cos \phi \qquad ...(6.1.7)$$

Projection of OQ on y-axis is

$$OS = OQ \sin \phi = r \sin \theta \sin\phi$$

$$\therefore \quad y = r \sin \theta \sin\phi \qquad ...(6.1.8)$$

Projection of OP on z-axis gives z coordinates of P.

Projection of OP on z-axis = OT = OP cos θ

$$\therefore \quad z = r \cos \theta \qquad ...(6.1.9)$$

Syllabus Topic : Scalar and Vector Field

Q.   Explain Scalar and Vector Field.

Ans. :

Scalar Field

- A scalar field associates a scalar value to every point in a space. It may be representing either a mathematical number or a physical quantity.

- Scalar fields are required to be coordinate independent means if any two observers using the same units will essentially have same value of the scalar field at the same absolute point in space regardless of their respective points of origin.

- For example: Temperature, or pressure distribution throughout the space.

Vector Field

- A vector field can be imagined by assigning a vector to individual points.

- A vector field in the plane can be regarded as collection of arrows with a given magnitude and direction attached to a point in the plane.

- Vector fields are often used to (i) to model the speed and direction of moving fluid throughout the space (ii) to represent the strength and direction of some force such as magnetic or gravitational force as it changes from point to point.
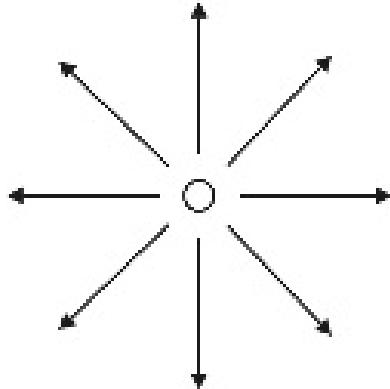
<div style="background:#d7ead3;border:3px solid black;padding:6px;text-align:center;font-weight:bold">
Syllabus Topic : Physical Significance of Gradient, Curl and Divergence
</div>

Q.   Explain physical significance of divergence.

Ans. :

Significance of Divergence
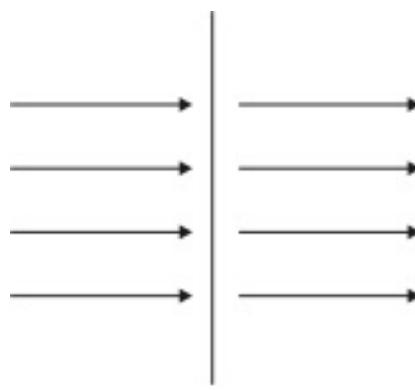
- It is known that divergence of vector field $\bar{A}$ is expressed as $\nabla \cdot \bar{A}$. It is given by,

$$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

- Clearly the divergence of a vector is a scalar. Also the divergence of a scalar cannot be obtained.

- The simple meaning can be expressed by considering net flux $\oint \bar{A} \cdot \bar{ds}$ of a vector field $\bar{A}$ from a closed surface S. The divergence of $\bar{A}$ is defined as the net outward flux per unit volume over a closed surface.

- In Fig. 6.3.2(a), at point O the divergence of vector field is positive as the vector spreads out which also represents source.

- In Fig. 6.3.2(b), the field is converging and hence divergence at point O is negative which represents sink. In Fig. 6.3.2(c), one can notice that divergence of vector field is zero.

- A water fountain, or a tyre which has just been punctured by a nail where air is expanding and forming a net outflow are some more examples which may be used to represent divergence.

(a) Positive divergence (source)   (b) Negative divergence sink

(c) Zero divergence (Incompressible)

Fig. 6.3.2

- For completeness, expressions for divergence in cylindrical and spherical co-ordinates are given as follows :

$$\text{div}\,\overline{A} \;=\; \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\overline{A}r\right) + \frac{1}{r}\frac{\partial \overline{A}_\phi}{\partial \phi} + \frac{\partial \overline{A}_z}{\partial z}$$

(For cylindrical co-ordinates)

$$\text{div}\,\overline{A} \;=\; \frac{1}{r}\frac{\partial}{\partial r}\left(r^2\,\overline{A}_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\overline{A}_\theta \sin\theta\right)$$

$$+\; \frac{1}{r\sin\theta}\cdot\frac{\partial \overline{A}_\phi}{\partial \phi}$$

(For spherical co-ordinates)

If $\nabla \times \overline{A} = 0$, the vector filed is said to be solenoidal.

Q.   What is curl of a vector ?.

Ans. :

curl of a vector

- Curl is simply defined as circulation per unit area where a closed path is vanishingly small. It is represented as $\nabla \times \overline{A}$ for a vector field $\overline{A}$.

Mathematically,

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

- Curl is a vector quantity as it has magnitude and direction both. In order to make its meaning clear, we consider the circulation of a vector field $\bar{A}$ around a closed path i.e. $\oint \bar{A} \cdot d\bar{l}$.

Syllabus Topic : Derivation of Maxwell's Four Equation

Q.   Derive point form of all Maxwell's equations.

Ans. :

Point form of all Maxwell's equations

- Let us assume an arbitrary surface 's' bounding an arbitrary volume V in a dielectric medium. For any dielectric medium, the total charge density present is the sum of free charge density (ρ) and polarized charged density (ρ_p).

- The total electric flux ψ crossing the closed surface is equal to the total charge enclosed by that surface (Gauss law of electrostatics)

$$\iint \bar{E} \cdot ds = \oint \bar{E} \cdot ds = \frac{1}{\varepsilon_0} \int_v (\rho + \rho_p)\, dV$$

Where q = ρV

$$\text{and} \qquad \rho_p = -\nabla \cdot \bar{P}$$

$$\oint \varepsilon_0 \bar{E} \cdot ds = \int_v (\rho - \nabla \cdot \bar{P})\, dV$$

- Using divergence theorem convert surface integral to volume integral

$$\oint \varepsilon_0 \bar{E} \cdot ds = \oint \nabla \cdot (\varepsilon_0 \bar{E})\, dV = \int_v (\rho - \nabla \cdot \bar{P})\, dV$$

$$\int_v \nabla \cdot (\varepsilon_0 \bar{E} + \bar{P})\, dV = \int_v \rho\, dV$$

- As electric displacement vector $\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$

$$\int_v \nabla \cdot \bar{D}\, dV = \int_v \rho\, dV$$

$$\text{Or} \int_v (\nabla \cdot \bar{D} - \rho)\, dV = 0 \qquad \nabla \cdot \bar{D} - \rho = 0$$

$$\nabla \cdot \bar{D} = \rho$$

- This is Point from of Maxwell's first equation.

- As the magnetic lines of force are closed (or go off to infinity), the number of magnetic lines of flux entering any surface is exactly the same as leaving. (The occurrence of an isolated magnetic pole has no physical significance).

$$\therefore \oint_{v} \bar{B} \cdot ds = 0$$

- Using Gauss divergence theorem, convert surface integral to volume integral.

$$\therefore \oint_{v} \bar{B} \cdot ds = \oint_{v} \nabla \cdot \bar{B} \, dv = 0$$

$$\therefore \nabla \cdot \bar{B} = 0$$

- This is point form of Maxwell's second equation.

- According to Faraday's law, electromagnetic force induced in a closed loop is negative rate of charge of the magnetic flux.

$$e = \frac{-d\phi}{dt} \qquad\qquad ...(6.4.1)$$

- Total magnetic flux on any arbitrary surface S

$$\phi = \oint_{s} \bar{B} \, ds \qquad\qquad ...(6.4.2)$$

$$\therefore e = \frac{-d}{dt}\left[\oint_{s} \bar{B} \cdot ds \right] = -\oint_{s} \left[\frac{d\bar{B}}{dt}\right] ds$$

- The electromotive force is the work done in carrying a unit charge around the closed loop.

$$\therefore e = \oint_{l} \bar{E} \cdot dl$$

$$\therefore \oint_{l} \bar{E} \cdot dl = -\oint_{s} \left(\frac{d\bar{B}}{dt}\right) ds$$

- By using Stoke's theorem contour integration can be converted to surface integration as

$$\oint_l \bar{E} \cdot d\ell = \oint_s \left(\bar{\nabla} \times \bar{E} \right) ds$$

$$\therefore \oint_s \left(\bar{\nabla} \times \bar{E} \right) ds = -\oint_s \frac{d\bar{B}}{dt} ds$$

$$\therefore \oint_s \left(\nabla \times \bar{E} + \frac{d\bar{B}}{dt} \right) ds = 0$$

$$\therefore \nabla \times \bar{E} = \frac{-d\bar{B}}{dt}$$

This is Maxwell's third equation.

- Consider the definition of work done by the magnetic field around the closed closed loop and

- Using Ampere's circuital law

$$\oint \bar{H} \cdot d\ell = I = \oint \bar{J} \cdot ds$$

Where $\bar{J}$ = current density

- Use Stoke's theorem to convert the contour integration to surface integrative.

$$\therefore \oint \bar{J} \cdot ds = \oint (\nabla \times \bar{H}) ds$$

$$\therefore \oint (\bar{J} - \nabla \times \bar{H}) ds$$

$$\therefore \bar{J} = \nabla \times \bar{H} \qquad\qquad ...(6.4.3)$$

- This equation needs to be verified for time varying field as its validity holds for steady state only. Because if it is correct then conservation of charge will be violated. The reason is very simple : div of curl is zero.

$$\text{div } J = \text{div}(\nabla \times \overline{H}) = 0$$

- This is in contrast with the continuity equation.

$$\text{div } \overline{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\text{or } \text{div } \overline{J} = -\frac{\partial \rho}{\partial t} \qquad \qquad ...(6.4.4)$$

- To correct this, Maxwell suggested the total current density needs an additional component i.e. $\overline{J}$.

$$\nabla \times \overline{H} = \overline{J} + \overline{J}'$$

- Now, if divergence is taken then using Equation (6.4.3).

$$\text{div}(\nabla \times \overline{H}) = \text{div}(\overline{J} + \overline{J}')$$

$$0 = \text{div}(\overline{J} + \overline{J}')$$

$$\text{div } \overline{J} = \text{div } \overline{J}'$$

- From Equation (6.4.4).

$$\text{div } \overline{J} = -\frac{\partial \rho}{\partial t}$$

- But from Maxwell's first Equation

$$\nabla \cdot \overline{D} = \rho$$

$$\therefore \text{div } \overline{J} = \frac{\partial}{\partial t}(\nabla \cdot \overline{D}) = \nabla \cdot \frac{\partial \overline{D}}{\partial t}$$

Hence, $\quad J' = \dfrac{\partial \overline{D}}{\partial t}$

- $\therefore$ Maxwell's fourth Equation is

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

- The additional term $\dfrac{\partial \overline{D}}{\partial t}$ is called Maxwell's correction and it is known as displacement current.

Q. Write integral form of all Maxwell's equations.

Ans. :

$$\text{As } \nabla \cdot \overline{D} = \rho$$

Take integration over complete volume V.

$$\therefore \int_V \nabla \cdot \overline{D} \, dv = \int_V \rho \, dv$$

Using Gauss's divergence theorem

$$\int_v \nabla \cdot \overline{D} \, ds = \oint_s \overline{D} \cdot ds$$

$$\therefore \oint_s \overline{D} \cdot ds = \int_V \rho \, dv$$

As $\quad \rho \, dv = q$

$$\oint \overline{D} \cdot ds = q$$

As $\qquad \nabla \cdot \overline{B} = 0$

Again using Gauss's divergence theorem and procedure adopted above one can arrive at

$$\oint_s \overline{B} \cdot ds = 0$$

Using differential form of Maxwell's Third Equation

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

Integrating over a surface

$$\int_s (\nabla \times \overline{E}) \cdot ds = -\int_s \frac{\partial \overline{B}}{\partial t} \, ds$$

Using Stoke's theorem, convert surface integral to line integral.

$$\oint_c \overline{E} \cdot dl = -\frac{\partial}{\partial t} \int \overline{B} \cdot ds$$

Using differential form of Maxwell's Equation

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

Using Stoke's theorem and procedure adopted in Section 6.5.3 one can arrive at

$$\oint_c \overline{H} \cdot dl = \int_c \left(\overline{J} + \frac{\partial \overline{D}}{\partial t} \right) ds$$

Syllabus Topic : Applications - Design of Antenna, Wave Guide, Satellite Communication

Q. Write short note on : Design of Antenna, Wave Guide, Satellite Communication.

Ans. :

Design of Antenna, Wave Guide, Satellite Communication

- In the present era of communication we are heavily dependent on various communication devices, like cell phone, radio, TV, satellite etc. Looking at the scope of communication field one can divide this into at least two part (i) Wired communication (ii) Wireless communication.

- For wired communications we need something like transmission line or waveguide. For wireless communication we require antenna to transmit or receive signals.

- In both the modes we badly require a complete knowledge of electromagnetism and Maxwell's equation. To carry electromagnetic wave when we make use of wired mode we make use of transmission lines or cables. For point to point transmission, the source energy must be directed for this purpose transmission line is use.

- Other way to make electromagnetic wave propagate is waveguide. A structure that can "guide" waves is called waveguide. The original and the most common meaning of waveguide is a hollow metal pipe used for guiding the

waves.

- The electromagnetic waves in such waveguides may be imagined as waves travelling down the guide in a zig-zag path as these waves are repeatedly reflected between opposite walls of guide. In order to analyze the mode propagation in the waveguide, we solve the Maxwell's equation.

- For wireless or satellite communications we essentially require antenna. While designing antenna, all the inputs like distance, medium, energy density, wavelength etc. are considered.

Solved Problems

Ex. 6.7.1

Two vectors are represented by

$$\bar{A} = 2\,\bar{a}_x + 2\,\bar{a}_y + 0\,\bar{a}_z \; ; \; \bar{B} = 3\bar{a}_x + 4\,\bar{a}_y - 2\bar{a}_z$$

Find $\bar{A} \times \bar{B}$ and show that $\bar{A} \times \bar{B}$ is at right angle to $\bar{A}$.

Soln. :

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 2 & 2 & 0 \\ 3 & 4 & -2 \end{vmatrix} = \bar{a}_x\,(-4-0) - \bar{a}_y\,(-4-0) + \bar{a}_z\,(8-6)$$

$$= -4\bar{a}_x + 4\bar{a}_y + 2\bar{a}_z$$

The vector $\bar{A} \times \bar{B}$ is at right angle to $\bar{A}$ then

$$(\bar{A} \times \bar{B}) \cdot \bar{A} = 0$$

$$\text{L.H.S} = (-4\,\bar{a}_x + 4\,\bar{a}_y + 2\bar{a}_z) \cdot (2\bar{a}_x + 2\,\bar{a}_y) = -8 + 8 = 0 = \text{R.H.S}$$

$\therefore \bar{A} \times \bar{B}$ is at right angle to $\bar{A}$.

Ex. 6.7.2

Given vectors

$$\bar{A} = 3\,\bar{a}_x + 4\,\bar{a}_y + \bar{a}_z, \quad \bar{B} = 2\,\bar{a}_y - 5\,\bar{a}_z$$

Find the angle between $\bar{A}$ and $\bar{B}$ using (i) dot product (ii) cross product.

Soln. :

The magnitudes of vectors are,

$$A = |\bar{A}| = \sqrt{3^2 + 4^2 + 1^2} = \sqrt{26}$$

$$B = |\bar{B}| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$$

i) Using dot product

$$\bar{A}\cdot\bar{B} = AB\cos\theta$$

Here the dot product is obtained using

$$\bar{A}\cdot\bar{B} = (3 \times 0) + (4 \times 2) + (1 \times -5) = 3$$

$$\therefore 3 = \sqrt{26}\,\sqrt{29}\cos\theta \quad \therefore \cos\theta = \frac{3}{\sqrt{26}\,\sqrt{29}} = 0.1093$$

$$\theta = \cos^{-1}(0.1093) = 83.73°$$

ii) Using cross product

$$\bar{A}\times\bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 3 & 4 & 1 \\ 0 & 2 & -5 \end{vmatrix} = \bar{a}_x(-20-2) - \bar{a}_y(-15-0) + \bar{a}_z(6-0)$$

$$= -22\,\bar{a}_x + 15\,\bar{a}_y + 6\,\bar{a}_z$$

We have $\bar{A}\times\bar{B} = AB\sin\theta\,\bar{a}_n$

Taking mod of both sides,

$$|\bar{A}\times\bar{B}| = AB\sin\theta$$

$$\text{or} \quad \sin\theta = \frac{|\bar{A}\times\bar{B}|}{AB} = \frac{\sqrt{-22^2 + 15^2 + 6^2}}{\sqrt{26}\,\sqrt{29}} = \frac{\sqrt{745}}{\sqrt{26}\,\sqrt{29}} = 0.994$$

$$\text{or} \quad \theta = \sin^{-1}(0.994) = 83.73°$$

Ex. 6.7.3

Find area of rectangle in z = 5 plane with −1 ≤ x ≤ 3 and 0 ≤ y ≤ 5.

Soln. :

As z = 5,   dz = 0,   ∴   ds = dx dy

The total area is,

$$S = \int ds = \int_0^5 \int_{-1}^3 dx\,dy$$

$$= [x]_{-1}^3\,[y]_0^5 = 4 \times 5 = 20\,m^2$$
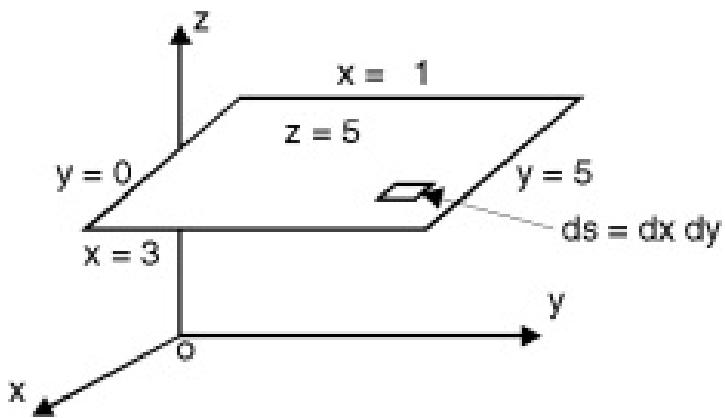
Fig. P. 6.7.3

Ex. 6.7.4

Find volume of a closed surface bounded by $0 \leq x \leq 2$, $0 \leq y \leq 1$ and $0 \leq z \leq 5$.

Soln. :

Differential volume is $dv = dx\, dydz$

∴ The total volume is,

$$V = \int dv = \int_0^5 \int_0^1 \int_0^2 dx\, dydz = [x]_0^2\, [y]_0^1\, [z]_0^5 = 2 \times 1 \times 5 = 10$$

m^3.

Ex. 6.7.7

Find the area of circular disk with radius of 2 m.

Soln. :

For simplicity place the circular disk with centre at origin in xy place. This circular disk is a part of cylinder in cylindrical system. It is horizontal, therefore

$z = $ constant $= 0$ or $dz = 0$

∴ differential area is $ds = r\, dr\, d\phi$

And total area is obtained by integrating this,

$$\therefore S = \int ds = \int_{0}^{2\pi}\int_{0}^{2} r\, dr\, d\phi = \left[\frac{r^2}{2}\right]_{0}^{2} [\phi]_{0}^{2\pi}$$

$$= 2 \times 2\pi = 4\pi \ (m^2)$$
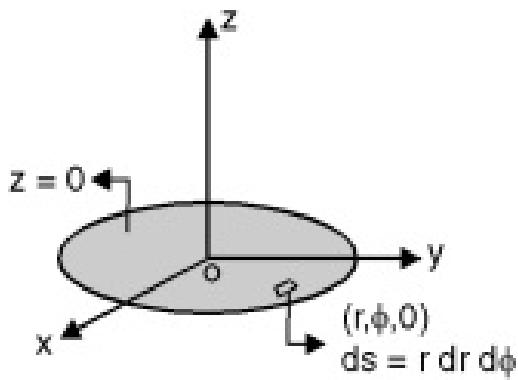
Fig. P. 6.7.7

Even if you consider this disk as a top plate of the cylinder, there is no problem, the answer will be same.

Ex. 6.7.8

Find the circumference of the circle with radius of 3 m.

Soln. :

Imagine the circle as a part of cylinder in cylindrical system either top or bottom of the cylinder. This is horizontal. For any point on the circle

$r = $ constant $= 3m$, and $z = $ constant $= 0$

$\therefore \quad dr = 0, \ dz = 0$

Then, $dl = r\, d\phi$

The total length i.e. circumference is,

$$= \int dl = \int_0^{2\pi} r \, d\phi = 3 \times [\phi]_0^{2\pi}$$

$$= 3 \times 2\pi = 6\pi \ (m) \qquad \ldots \text{since } r = 3$$

Fig. P. 6.7.8

Ex. 6.7.9

Find area of the cylindrical surface with height = 2 m and radius = 1 m.

Soln. :

For the cylindrical surface,

$$r = \text{constant} = 1 \text{ m.}$$

$$\therefore dr = 0 \quad \text{then} \quad ds = r \, d\phi dz$$

And $\qquad S = \int ds = \int_0^{2} \int_0^{2\pi} r \, d\phi dz$

$$= 1 \times [\phi]_0^{2\pi} \times [z]_0^2$$

$$= 1 \times 2\pi \times 2 = 4\pi \ (m^2)$$
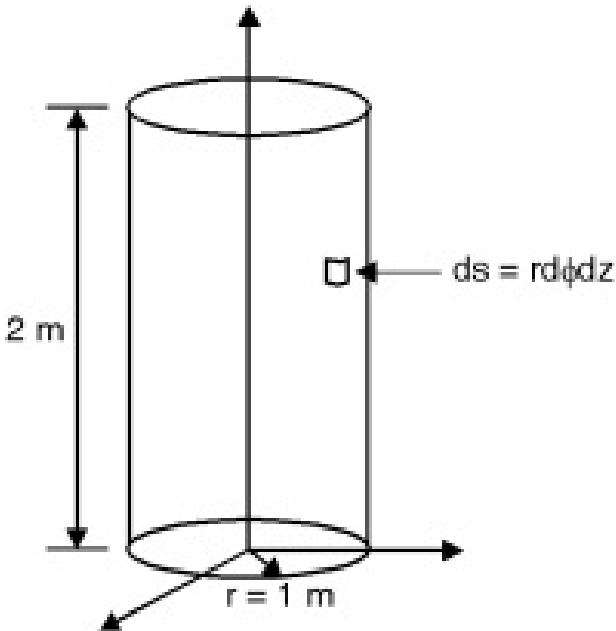
Fig. P. 6.7.9

Ex. 6.7.10

Use of the cylindrical coordinate system to find the area of a curved surface on the right circular cylinder of radius 2 m, height 8 m and $45° \leq \phi \leq 90°$.

Soln. :

The given surface is a part of cylinder with r = 2m. then

$$dS = r\, d\phi dz = 2\, d\phi dz$$

The total area is obtained by integrating it.

$$S = \int_S dS = 2 \int_0^8 \int_{\pi/4}^{\pi/2} d\phi\, dz$$

$$= 2 \times (\phi)_{\pi/4}^{\pi/2} \cdot (z)_0^8$$

$$= 2 \times \frac{\pi}{4} \times 8 = 4\pi \ (m^2)$$
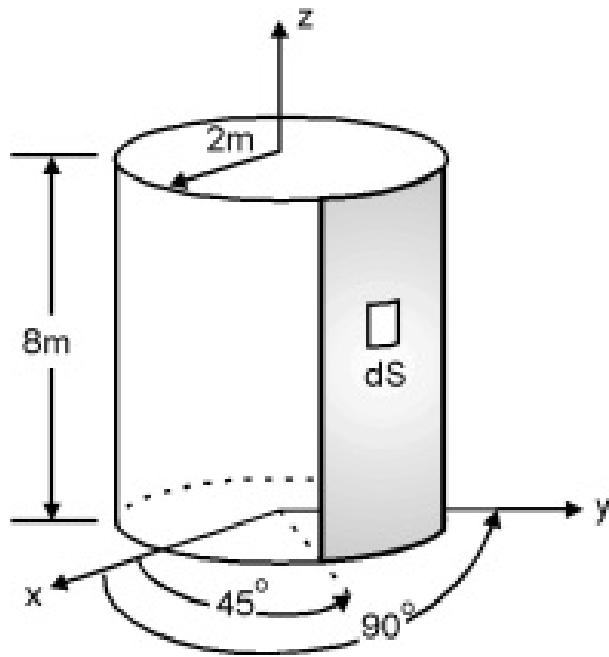
Fig. P. 6.7.10

Ex. 6.7.29

If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ at the point $(1, -2, -1)$.

Soln. :

The gradient of ϕ is given by,

$$\nabla\phi = \frac{\partial\phi}{\partial x}\,\bar{a}_x + \frac{\partial\phi}{\partial y}\,\bar{a}_y + \frac{\partial\phi}{\partial z}\,\bar{a}_z$$

$$\frac{\partial\phi}{\partial x} = 6xy; \quad \frac{\partial\phi}{\partial y} = 3x^2 - 3y^2 z^2; \quad \frac{\partial\phi}{\partial z} = -2y^3 z.$$

Thus, $\quad \nabla\phi = 6xy\,\bar{a}_x + (3x^2 - 3y^2 z^2)\,\bar{a}_y - (2y^3 z)\,\bar{a}_z$ at $(1, -2, 1)$,

$$\nabla\phi = 6 \times 1 \times (-2)\,\bar{a}_x + [3 \times (1)^2 - 3(-2)^2(-1)^2]\,\bar{a}_y + [2(-2)^3(-1)]\,\bar{a}_z$$

$$= -12\,\bar{a}_x - 9\,\bar{a}_y - 16\,\bar{a}_z$$

Ex. 6.7.30

If $\bar{A} = x^2 z\,\bar{a}_x - 2y^2 z^2\,\bar{a}_y + xy^2 z\,\bar{a}_z$, find $\nabla \cdot \bar{A}$ at the point $(1, -1, 1)$.

Soln. :

The divergence of $\bar{A}$ is given by,

$$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Here, $\quad \dfrac{\partial A_x}{\partial x} = 2xz; \dfrac{\partial A_y}{\partial y} = -4yz^2; \dfrac{\partial A_z}{\partial z} = xy^2$

Then $\quad \nabla \cdot \bar{A} = 2xz - 4yz^2 + xy^2$

At $(1, -1, 1)$, $\nabla \cdot \bar{A} = 2(1)(1) - 4(-1)(1)^2 + (1)(-1)^2 = 7$

Ex. 6.7.31

Find the divergence of the vector function $\bar{A} = x^2\,\bar{a}_x + x^2 y^2\,\bar{a}_y + 24\,x^2\,y^2\,z^2\,\bar{a}_z$

Evaluate the integral of $\nabla \cdot \bar{A}$ throughout the volume of a unit cube centered at the origin.

Soln. : $\quad$ For $\bar{A} = A_x\,\bar{a}_x + A_y\,\bar{a}_y + A_z\,\bar{a}_z$

$$\text{Div. } \bar{A} = \nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Here, $\quad \dfrac{\partial A_x}{\partial x} = 2x; \dfrac{\partial A_y}{\partial y} = 2x^2 y; \dfrac{\partial A_z}{\partial z} = 72x^2 y^2 z^2$

$\therefore \nabla \cdot \bar{A} = 2x + 2x^2 y + 72x^2 y^2 z^2$

Since the unit cube is centered at the origin, the x, y, z coordinates varies each from $-(1/2)$ to $(1/2)$. Now the volume integral of $\nabla \cdot \bar{A}$ is

$$\int_V (\nabla \cdot \bar{A})\, dv = \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} (\nabla \cdot \bar{A})\, dx\, dy\, dz$$

$$= \int\limits_{-1/2}^{1/2} \int\limits_{-1/2}^{1/2} \int\limits_{-1/2}^{1/2} (2x + 2x^2y + 72x^2y^2z^2)\, dx\, dy\, dz$$

$$= 2\left[\frac{x^2}{2}\right]_{-1/2}^{1/2} [y]_{-1/2}^{1/2} [z]_{-1/2}^{1/2} + 2\left[\frac{x^3}{3}\right]_{-1/2}^{1/2}\left[\frac{y^2}{2}\right]_{-1/2}^{1/2} [z]_{-1/2}^{1/2}$$

$$+ 72\left[\frac{x^3}{3}\right]_{-1/2}^{1/2}\left[\frac{y^3}{3}\right]_{-1/2}^{1/2}\left[\frac{z^3}{3}\right]_{-1/2}^{1/2}$$

$$= 41.67 \times 10^{-3}$$

Ex. 6.7.32

Determine the 'curl' of these vector field.

(i) $\overline{A} = \overline{a}_x (2x^2 + y^2) + \overline{a}_y (xy - y^2)$ (ii) $\overline{A} = yz\, \overline{a}_x + 4xy\, \overline{a}_y + y\, \overline{a}_z$

Soln. :

(i)

$$\nabla \times \overline{A} = \begin{vmatrix} \overline{a}_x & \overline{a}_y & \overline{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2x^2 + y^2 & xy - y^2 & 0 \end{vmatrix}$$

$$= \overline{a}_x (0 - 0) - \overline{a}_y (0 - 0) + \overline{a}_z (y - 2y) = -y\, \overline{a}_z$$

(ii)

$$\nabla \times \overline{A} = \begin{vmatrix} \overline{a}_x & \overline{a}_y & \overline{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yz & 4xy & y \end{vmatrix} = \overline{a}_x (1 - 0) - \overline{a}_y (0 - y) + \overline{a}_z (4y - z)$$

$$= \overline{a}_x + y\, \overline{a}_y + (4y - z)\, \overline{a}_z$$

Charged Particle in Electric & Magnetic Fields

Q.   Describe Motion of Electron in Electric Field (Parallel).

Ans. :

Motion of Electron in Parallel Electric Field

- Motion of electron in electric field such that electron is moving parallel to external electric field.
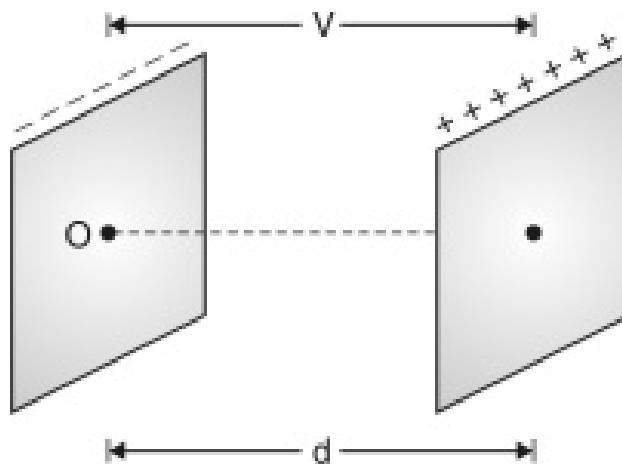
Fig. 7.1.1

- As electron is a negatively charged particle with e = 1.6 × 10^{-19} C and mass m_e = 9.1 × 10^{-31} kg ; for the

arrangement shown in Fig. 7.1.1 we can derive the following.

- In the Fig. 7.1.1 two plates are separated by spacing 'd' and potential difference V. The plate on RHS is positively charged.

- The electric field intensity is

$$E = \frac{V}{d} \qquad \qquad ...(7.1.1)$$

  and the acceleration experienced by electron entering at point '0' is

  $$F = ma = eE$$

  $$a = \left(\frac{e}{m}\right)E \qquad \qquad ...(7.1.2)$$

- The terminal velocity of an electron that has travelled through a potential difference V is given by

$$\frac{1}{2}m\upsilon^2 = eV$$

$$\therefore \quad \upsilon = \sqrt{\frac{2eV}{m}} \qquad \qquad ...(7.1.3)$$

Syllabus Topic : Motion of Electron in Electric Field (Perpendicular)

Q.   Describe motion of electron is perpendicular to the applied electric field.

Ans. :

Motion of Electron in Electric Field (Perpendicular)

- In Fig. 7.2.1 we have two plates with length 'l' and separated by a distance 'd'. The upper plate is having positive charge and the lower plate is having negative charge.
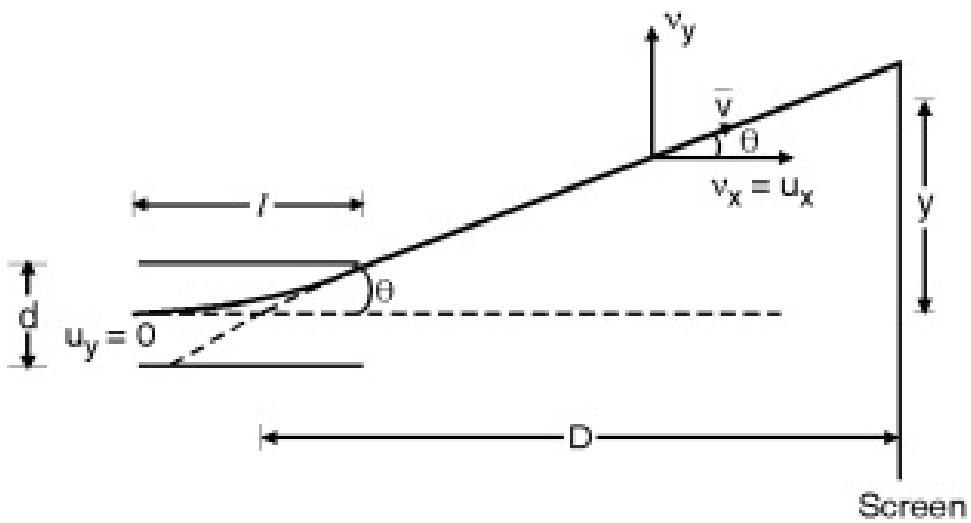
Fig. 7.2.1

- The potential difference between these two plates is V. An electron is made to enter this perpendicular field with initial velocity u_x ($u_y = 0$) due to accelerating potential V_A. Electron will experience perpendicular electric field by a force F_y.

- The path of electron will become parabolic till it takes an exist and later move on straight line path with velocity v, (with $u_x = v_x$) and hit the screen at deflection 'y' as shown in Fig. 7.2.1.

- The force acting on Y direction is

$$F_y = ma_y = e\left(\frac{E_d}{d}\right)$$

as $\quad F_y = eV_d = (V_d = \text{deflecting voltage})$

$$\therefore a_y = \frac{eE_y}{m}$$

Now $u_y = 0$, displacement at any instant t in the y-direction is $\qquad$ given by

$y = \frac{1}{2} a_y t^2 = \frac{1}{2} \left(\frac{eE_y}{m}\right) t^2$

- To find t consider the following :

  As velocity in x-direction is $u_x = v_x$

  $$\therefore x = u_x t$$

(small illegible equations in left margin)

  $$y = k\,x^2 \qquad ...(7.2.1)$$

  $\therefore$ An electron travels a parabolic path in transverse electric field.

- After exit from perpendicular electric field, electron moves an straight line path at angle 'θ'.

  $\therefore$ From Fig. 7.2.1

(small illegible equations in left margin)
Slope = $\tan\theta$ ...

  $$\text{as } v_y = u_y + a_y t = a_y t \qquad (\text{as } u_y = 0)$$

(small illegible equations in left margin)

  $$\text{By taking KE } = \frac{1}{2} m\, u_x^2 = e\, V_A$$

  $$u_x^2 = \frac{2e\, V_A}{m}$$

  $$\therefore y = D \cdot \left(\frac{e}{m}\right) \cdot \frac{l}{E_y \cdot \left(\frac{2e}{m}\right)}\, V_A$$

  $$= \frac{D \cdot E_y \cdot l}{2 \cdot V_A}$$

  $$\text{Now } E_y = \frac{V_d}{d}$$

  $$\therefore y = \frac{D}{2} \cdot \left(\frac{V_d}{V_A}\right) \cdot \frac{l}{d}$$

Syllabus Topic : Motion of Electron in Magnetic Field(Longitudinal and Transverse)

Q. Write a note on longitudinal and transverse magnetic field.

Ans. :

Motion of Electron in Magnetic Field (Longitudinal)

- Consider the fact that magnetic force experienced by charged particle when it enters a magnetic field B with velocity v is given by

$$\bar{F} = q\left(\bar{B} \times \bar{v}\right)$$

$$= q\,B\,v\,\sin\theta$$

- For longitudinal magnetic field, $\theta = 0$ or $180°$ hence $\sin\theta = 0$ and $F = 0$. As magnetic force is zero, motion of charged particle remains unaffected.

Motion of Electron in Magnetic Field (Transverse)

- When an electron or any charged particle travels through magnetic field of strength B with velocity v then the force is given by

$$\bar{F}_m = q\left(\bar{B} \times \bar{v}\right)$$

$$= q\,B\,v\,\sin\theta$$

- If B and v are in same direction then $\theta = 0$ $\therefore F_m = 0$

- If B and v are perpendicular to each other (transverse field) then $\theta = 90$

$$\therefore \sin\theta = 1$$

$$\therefore F_m = q\,B\,v$$

- Since B, v, q = e all are constants in magnitude, $\bar{F}_m$ is constant in magnitude and perpendicular to the direction of motion.

- Due to this type of force, electron will be continuously deflected by Fleming's left hand rule and finally results in motion in a circular path with constant speed within the

transverse magnetic field. The force is similar to centripetal force. If this circular path has radius R, then

$$F_n = \frac{mv^2}{R}$$

$$\text{as } F_n = Bev = \frac{mv^2}{R}$$

$$\therefore \quad R = \left(\frac{m}{e}\right) \cdot \frac{v}{B} \qquad \qquad \dots(7.3.1)$$

Syllabus Topic : Magnetic Deflection

Q.   Explain the term magnetic deflection.

Ans. :

Magnetic Deflection

- If an electron enters a magnetic field at an angle θ to its initial velocity, the particle describes a helical path in the field. The axis of helix will be the direction of magnetic field as shown in Fig. 7.4.1.
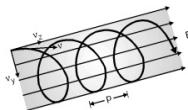
Fig. 7.4.1 : Electrostatic focussing

- The pitch p of the helix is given by $P = \frac{2\pi}{B}\left(\frac{m}{e}\right) v \cos\theta$ \qquad ...(7.4.1)

- If T is the time period to complete one revolution then

$$T = \frac{\text{Circumference}}{v} = \frac{2\pi R}{v}$$

Using Equation (7.4.1)

$$T = \frac{2\pi}{v}\left(\frac{m}{e}\right)\cdot\frac{v}{B}$$

$$T = \frac{2\pi}{B}\left(\frac{m}{e}\right) v \qquad \dots(7.4.2)$$

- When field is acting normally but over a limited extension then the path will define an arc for a limited distance l or

till the electron is in the field.

- Once out of magnetic field, it will follow a tangential path and reach the screen which is at distance of D from centre of magnetic field.
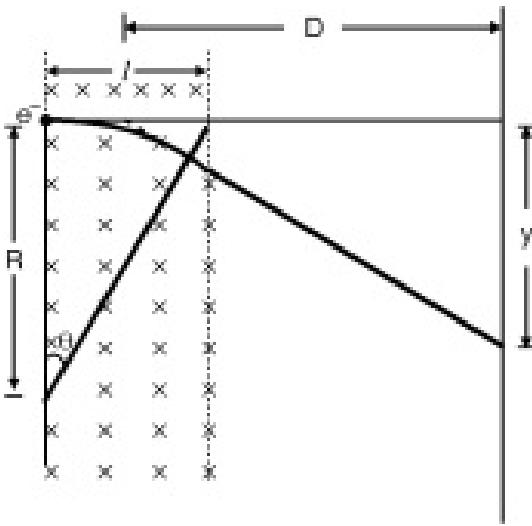
Fig. 7.4.2

The deflection y on screen is given by

$$y = D/B \sqrt{\frac{e}{2m \cdot V_A}} \qquad \qquad ...(7.4.3)$$

Syllabus Topic : Motion of Electron in Crossed Field

Q.   Write short note on : Motion of electron in crossed field.

Ans. :

Motion of Electron in Crossed Field

- Consider an electron moving with an initial velocity v entre in a crossed electric and magnetic field. The main feature of cross field is to cancel the deflection caused by electric and magnetic fields.

- Within crossed electric and magnetic fields, if the electron

is made to travel along x-direction then due to electric field alone it will defect along positive y direction.

- Whereas due to magnetic field alone it will deflect along negative y direction.

- Now if force acting on electron by electric and magnetic fields are simultaneously same, then there will not be any deflection.

as force due to electric field

$$F_E = eE$$

and due to magnetic field

$$F_E = Bev$$

$$\therefore \quad \cancel{e}E = B\cancel{e}v$$

$$v = \frac{E}{B} \qquad \qquad \ldots(7.5.1)$$

Syllabus Topic : Velocity Selector and Velocity Filter

Q. Write short note on : Velocity selector and velocity filter.

Ans. :

Velocity Selector and Velocity Filter

- As seen in case of crossed field,

$$v = \frac{E}{B}$$

- Using this formula one can design an arrangement which makes selection of monoveloicty charged particles.
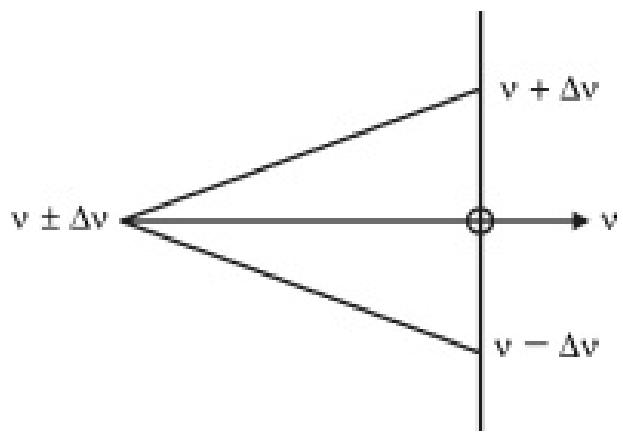
Fig. 7.6.1

- As shown in Fig. 7.6.1 if there is a beam of charged particles with velocity $v \pm \Delta v$ (i.e. mixed velocities to same extent) then particles will be filtered for $v + \Delta v$ and $v - \Delta v$ as the given values of E and B are such that only those charges with velocity 'v' will be able to go on straight path (selection) where as $v + \Delta v$ will experience higher F_E hence it will go above straight path and $v - \Delta v$ will experience higher F_m hence it will go below straight path (filtration).

Syllabus Topic : Electrostatic Focussing, Electron Refraction, Bethe's Law

Q. Derive Bethe's law for electron refraction. (May 2013, May 2016)

Ans. :

Bethe's law for electron refraction
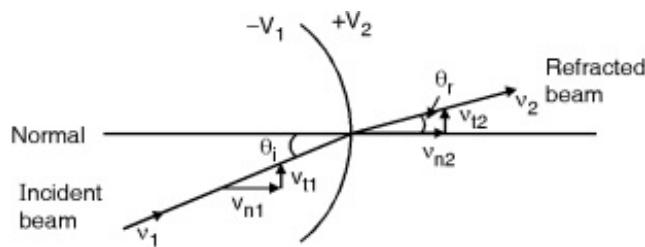
Fig. 7.7.2 : Refraction of a beam

- In Fig. 7.7.2, we have an equipotential surface. It is considered as a boundary across which the potential V_1 abruptly changes to potential V_2.

- Let an electron through a region on LHS of equipotential surface with a uniform velocity v_1 and enter the region of RHS.

- Because of this, it velocity alters. Since the force acts in a direction normal to the surface, its normal component of velocity that is altered. If $V_2 > V_1$, $v_{n2} > v_{n1}$ and if $V_1 > V_2$, $v_{n2} < v_{n1}$. We select $V_2 > V_1$. The electron path is therefore bent nearer to the normal.

- Since there is no force acting tangentially, tangential component will remain the same.

$$\therefore \quad v_{t1} = v_{t2} \qquad \qquad \ldots(7.7.2)$$

From Fig. 7.7.2

$$v_{t1} = v_1 \sin \theta_i, \qquad \qquad v_{t2} = v_2 \sin \theta_r$$

$$\therefore \quad v_1 \sin \theta_i = v_2 \sin \theta_r$$

$$\therefore \quad \frac{\sin \theta_i}{\sin \theta_r} = \frac{v_2}{v_1} \qquad \qquad \ldots(7.7.3)$$

Where v_2 = final velocity after leaving surface

v_1 = initial velocity with which electron approches

- Equation (7.7.3) is identical of the Snell's law in optics which describes refraction. This is also referred as Bethe's law.

Q. Write a short note on electrostatic focussing. (Dec. 2013)

Ans. :

Electrostatic Focussing

- As we know, in geometrical optics, focussing of light ray by a lens is due to phenomenon of refraction more popularly Snell's law.

- Let us find out how electron optics provided focussing of electron beam. Before we proceed, let us take idea of "equipotential surface".

- Equipotential surface is defined as a surface in an electric field where the electric potential remains the same.

- Consider Fig. 7.7.1 where an electron is at rest and placed in an electric field produced by two plates A and B.
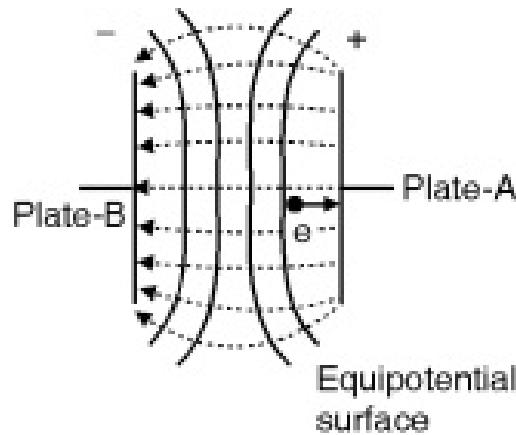
Fig. 7.7.1 : Electric field between two plates

- If an electron at rest placed in an electric field produced by two plates A and B, the force experienced by electron is

$$F = -eE$$

- The negative sign indicates that force acts in the opposite direction to that of field. This discussion is common but

valid only if field is uniform (i.e. within the area of plates A and B). When areas near boundaries of plates are concerned, the field is no more uniform.

- The field intensity is low at both the ends as shown with spreading of field lines. In the diagram we have equipotential surfaces shown. The force on electron is normal to the equipotential surface.

Syllabus Topic : Magnetic Focussing

Q.   Define magnetic focussing.

Ans. :

Magnetic Focussing

- Focussing can also be achieved using magnetic field produced by solenoid etc. called magnetostatic focussing.

- Magnetostatic focussing is divided in to two categories viz (a) longitudinal magnetic field focussing (b) transverse magnetic field focussing.

Syllabus Topic : Cathode Ray Tube (CRT)

Q.   With neat diagram explain construction and working of CRT. (May 2014, May 2015)

OR

Q.   Explain the principle, construction and working of CRT with neat diagram. (Dec. 2015)

Ans. :

A cathode ray tube is a specially designed vacuum tube in which an electron beam controlled by electric or magnetic fields are used to visual display of input electrical signal on the screen

which is coated with fluorescent materials.
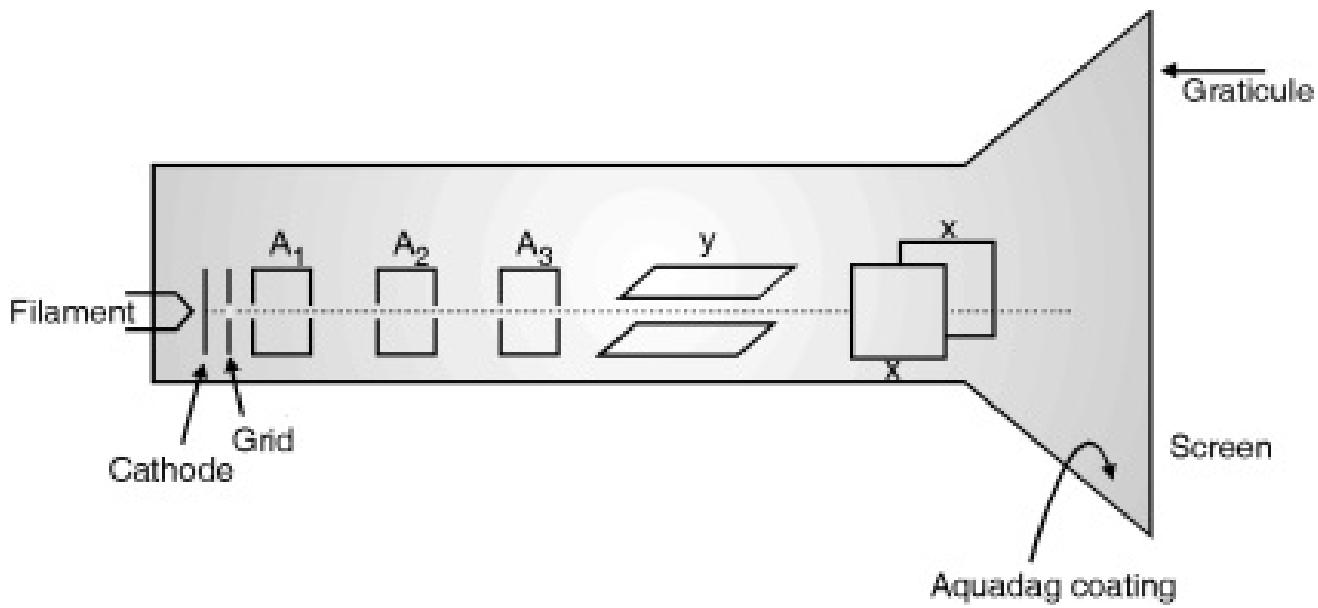
Fig. 7.12.1 : Schematic diagram of CRT

A CRT is a complex arrangement it is described by various parts like,

    (a)   Electron gun   (b)   Deflection system

    (c)   Fluorescent screen   (d)   Glass envelope

(a) Electron gun

The focussed and accelerated electron beam is given out by the electron gun.

It consists of a heater, a cathode, a grid, a pre accelerating anode, a focussing anode and an accelerating anode.

Cathode

Indirectly heated cathode which is usually coated with barium and strontium oxide which generates thermionic electrons.

Control gird

- Usually a nickel cylinder with a centrally located hole, co

axial with CRT axis. We know that intensity of electron beam depends upon the number of electrons emitted from the cathode.

- The control grid with its negative bias, controls the number of electrons emitted from the cathode and hence the intensity is controlled by the grid.

Preaccelerating anode and accelerating anode

The electrons emitted from the cathode and passing through the grid are accelerated by the high positive potential which is applied to pre accelerating and accelerating anodes. Electrons coming out from focussing anode are further accelerated through accelerating anode.

Focussing anode

Focussing of electron beam is achieved through focussing anode. Focussing anode has its potential around 500 V, which is adjustable. The potential of focussing anode is smaller than accelerating anode. Electrostatic focussing is used for focussing.*

(b) Deflection system

On refering article "Pre requisite" and points 6, 7 and 8 over there we get a general picture of electrostatic deflection. There are two pairs of strictly parallel metal plates, two of them are parallel to X-axis called y plates as the causes deflection along Y-axis where as other two which are parallel to Y-axis and called x-plates as they causes deflection along X-axis, as shown in Figs. 7.12.2(a) and (b). The deflected beam generates as bright spot on the screen.

Table 7.12.1 : Location of spot on screen

1.	Upper y-plate positive	On positive y-axis
2.	Lower y-plate positive	On negative y-axis
3.	LHS x-plate positive	On negative x-axis
4.	RHS x-plate positive	On positive x-axis
5.	Upper y-plate and RHS x-plate positive with equal potential	In the first quarter at 45°

* For large screens like TV or RADAR, electromagnetic focussing is used.

As shown in 'pre requisite' point-8 the deflection on Y-axis is given by

$$y = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{V}{V_A} \qquad \qquad ...(7.12.1)$$

Where y = Deflection on y-screen

l = Length of electric field applied

d = Spacing between two y-plates

D = Distance of point of screen apparent origin to the screen (apparent origin is at the center of deflection plates)

y plates.

Screen

(a)

X plates

(b)

y

x

x

y

Screen
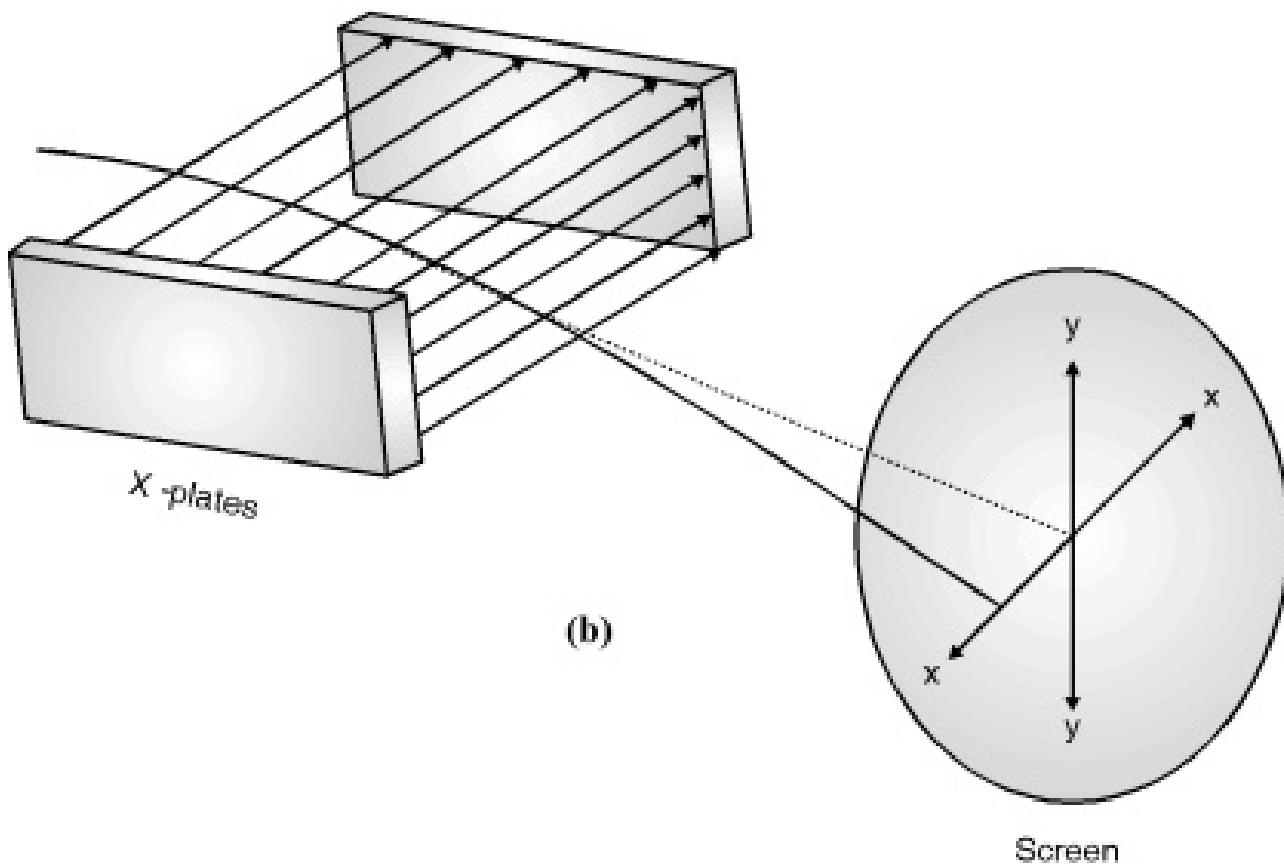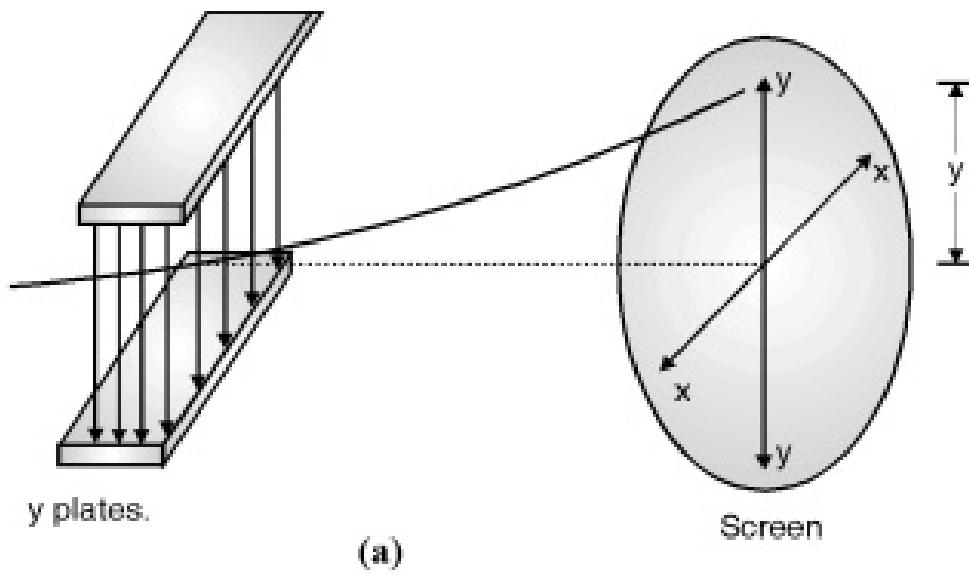
Fig. 7.12.2 : Deflection in CRT

From Equation (7.12.1) it is clear that,

(1) For a given acceleration potential and for particular dimensions of CRT, the deflection of the electron beam is directly proportional to the deflecting voltage.

$$y \propto V$$

∴ CRT may be used as indicating device.

(2) Since deflecting voltage is usually a time varying, image on the screen will follow a variation of the deflecting voltage in a linear manner.

Deflection sensitivity of CRT

It is defined as the deflection of the screen per unit deflection voltage.

$$\therefore \text{Deflection sensitivity} = S = \frac{y}{V}$$

$$\therefore S = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{1}{V_A} \qquad \ldots (7.12.2)$$

Unit of S is $\frac{m}{V}$

Deflection factor

It is defined as reciprocal of S

$$\text{i.e.} \quad G = \frac{1}{S} = \frac{2}{D} \cdot \frac{d}{l} \cdot V_A \qquad \ldots (7.12.3)$$

(c) Fluorescent screen

- The interior surface of circular front face of the CRT is coated with a thin translucent layer of phosphors.

- This consists of very pure inorganic crystalline phosphor crystal and with very small quantity of other elements like silver, manganese, chromium etc, called activators are found affecting characteristics of phosphors such as luminous efficiency, spectral emission etc.

- When the high energy electron beam strikes the phosphor coating, if glows at that point and continues to glow for a short period of time even after the electron beam is discontinued. This is known as persistence or phosphorescence.

- Aluminium in form of a thin foil is usually deposited on non-viewing side of the phosphor. It has following effects :

  (i) Fast moving electron when strike the screen they generate heat. It works as heat sink.

  (ii) The scattering of light is reduced and hence brightness is increased.

  (iii) The electron striking the screen release secondary electrons. Though acquadag is used, under certain conditions they remain on the screen and a negative voltage is build up. Aluminium layer prevents such charge build up.

Acquadag

When fast moving electrons strikes the screen, secondary electrons are released. These secondary electrons are collected by an aqueous solution of graphite called acquadag which is connected to the second anode. Secondary emitted electrons must be collected otherwise CRT screen will loose the state of electrical equilibrium.

Syllabus Topic : Cathode Ray Oscilloscope (CRO)

Q.   Explain construction and working of cathode ray oscilloscope. (Dec. 2014)

Ans. :
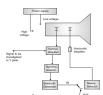
Fig. 7.14.1 : A diagram of general purpose CRO

CRO is an important instrument in any laboratory where different current and voltage signals are studied. Even non electrical quantities once converted to electrical equivalent using transducers can be visualized using CRO.

Let us see a block diagram of a general purpose CRO, with important sections on it.

The Fig. 7.14.1 has details of important section of a general purpose CRO. Let us discuss their role step by step.

CRT

- CRT as mentioned in Section 7.13 is heart of any CRO. It generates electrons, make them accelerated, focuss them and cause deflection as per requirement with the help of other electronic circuits.

- The signal under investigation needs to reach on screen and it should develop visual image of such signal.

- This is possible if electron beam is moving on X-Y plane on screen that is it should move along X and Y axes simultaneously. This is done with following circuit sections

Vertical deflection system

- The signals to be examined is usually applied to Y deflection plates through an amplifier or attenuator. The signals if not strong enough to produce measurable deflection on the CRT screen, an amplifier is essential.

- At the same time when high voltage signals are to be examined, they must be alternated to bring them within the range of vertical amplifier.

- The vertical amplifier output is also applied to the synchronizing amplifier stage through synchronizer

selector switch when internal Horizontal sweep circuit is to be triggered.

Horizontal deflection system

- The function of this stage is to spread the signal along x-axis. Let us see it through following diagram shown in Fig. 7.14.2.

- If a sine wave is applied to Y plates it is expected that on screen a sine wave is observed. But if there is no signal on X-deflection plate, we will get the image on screen as shown in Fig. 7.14.2 that is a vertical line.

Sine wave
given to Y-plate

Effect on
screen
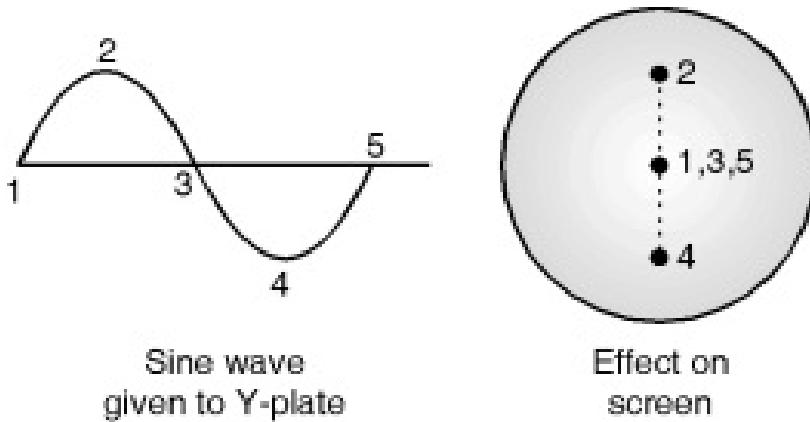
Fig. 7.14.2 : No x-plate signal applied

- This problem is sorted out by using horizontal deflection system which is responsible to provide a sweep signal which provides a time base. The horizontal plates are supplied through an amplifier but they can be fed directly when voltages are of sufficient magnitudes.

    Signal to the horizontal (X) plates can be supplied by two different ways.

    (i)   Using external signal

    (ii) Using internal signal generator

- This choice is available on CRO panel as sweep selector switch when the sweep selector switch is in the 'internal' position, the horizontal amplifier receives an input form the saw tooth sweep generator which is triggered by the synchronizing amplifier.

Sawtooth Sweep Generator

The voltage signal given to horizontal deflection plates should have following characteristics :

- The electron beam should have linear displacement with voltage applied to x-plates.
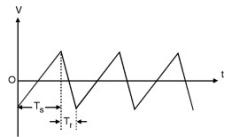
Fig. 7.14.3 : Typical sawtooth waveform

- At the end of the horizontal motion, the beam should return back starting point to repeat the motion.

- All the qualities are possessed by a saw tooth wave or ramp voltage as shown in Fig. 7.14.3.

- Ramp voltage increases linearly with time and decrease rapidly. It can be divided into two parts that is T_S = Sweep time or trace time and T_R = Retrace time, ($T_S >> T_R$ T_S). Retrace time is also called as flyback time.

Blanking pulse

- The retrace should be eliminated or blanked out that is the beam when goes back to its start point form right to left it should remain invisible.

- If there is no voltage applied across Y deflection plates, the image created on screen is shown in Fig. 7.14.4.
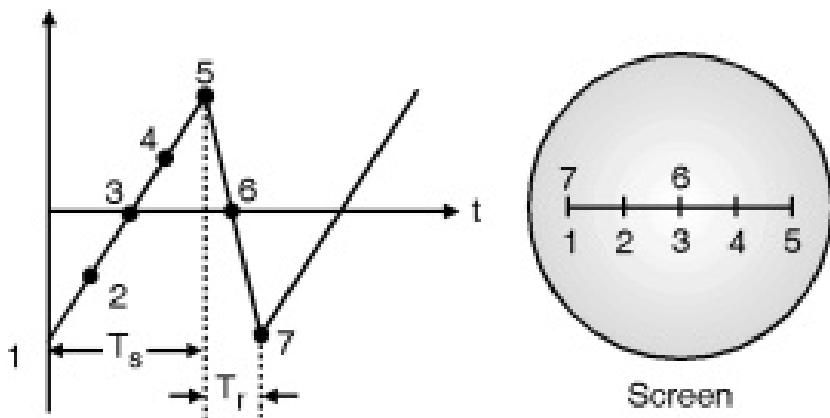
Fig. 7.14.4 : Ramp voltage and its effect on CRO screen

- It is easy to see that the length of the trace on the screen is measure of the period of the oscillator frequency in seconds. The x-axis of the screen not only denotes the amount of horizontal deflection but also the time elapsed.

- Due to this reason, the ramp voltage is also known as time base. Therefore the x-axis on the screen is calibrated in milliseconds and microseconds.

- Since signals of different frequencies are to be observed with the CRO, the sweep rate must be adjustable. We can change the sweep rate in steps. The front panel control for this adjustment is marked time/div or sec/div.

Display of Signal on Screen

How a signal gets displayed on CRO screen on Fig. 7.14.5 with the help of two cases,

(i) where $T_{signal} = T_{sweep}$.

(ii) $T_{signal} = 2\,T_{sweep}$

Case - I : T$_{signal}$ = T$_{sweep}$

CRO screen

Input voltage applied to Y-plate

Sawtooth sweep voltage applied to X-plates
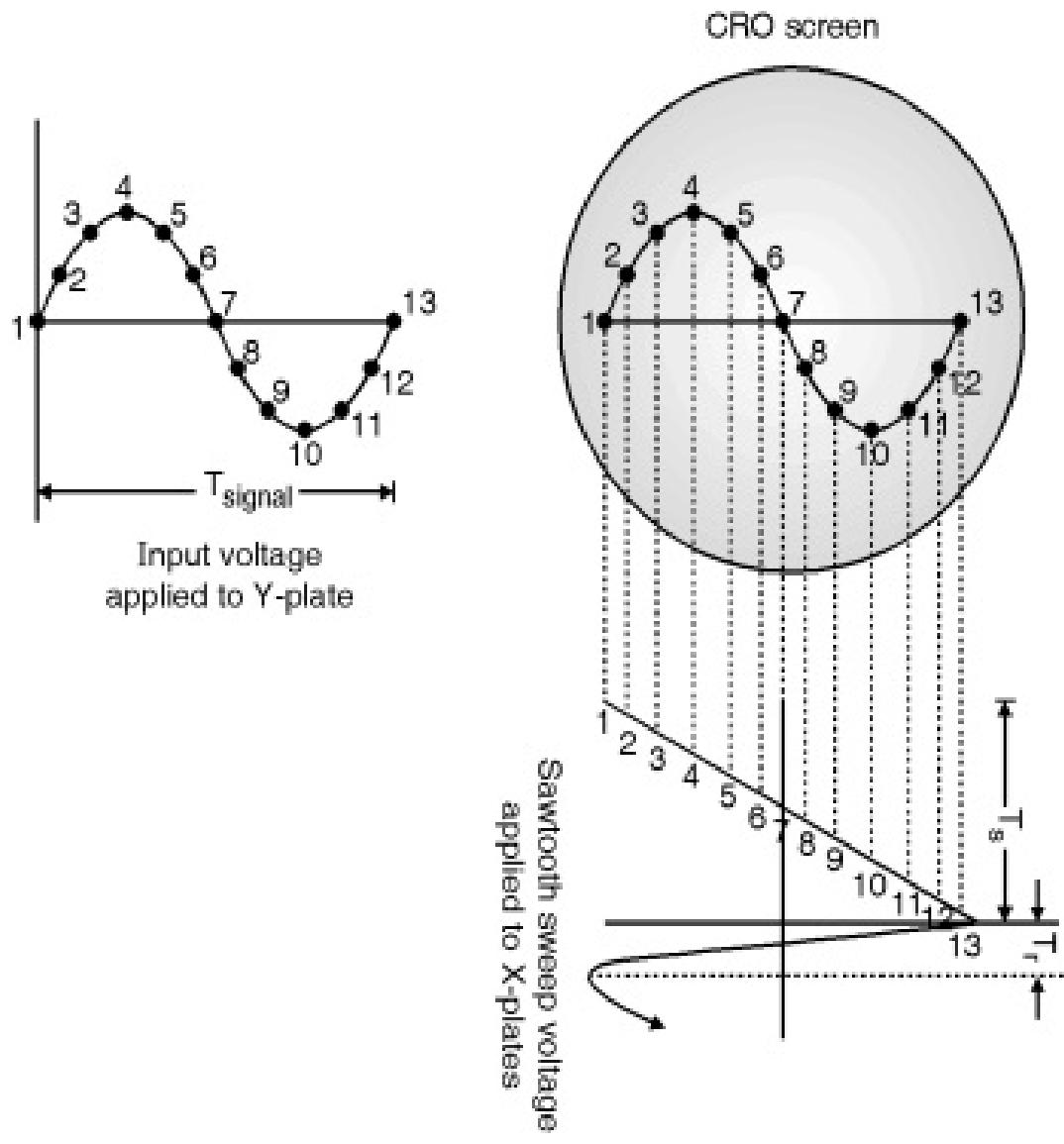
Fig. 7.14.5(a) : When T$_{signal}$ = T$_{sweep}$

Case – II : T$_{signal}$ = T$_{sweep}$

As T$_{signal}$ = 2 T$_{sweep}$, only half the signal is visible on CRO screen.

Above mentioned cases explain that when the signal applied to Y – plate and time base signal on x-plate are considered simultaneously. The beam will experience two force acting in perpendicular direction. The deflection of the beam at any instant occurs along the direction of the resultant of the two

forces. Both the cases give a complete brief explanation as how a signal gets it's image constructed on CRO screen.
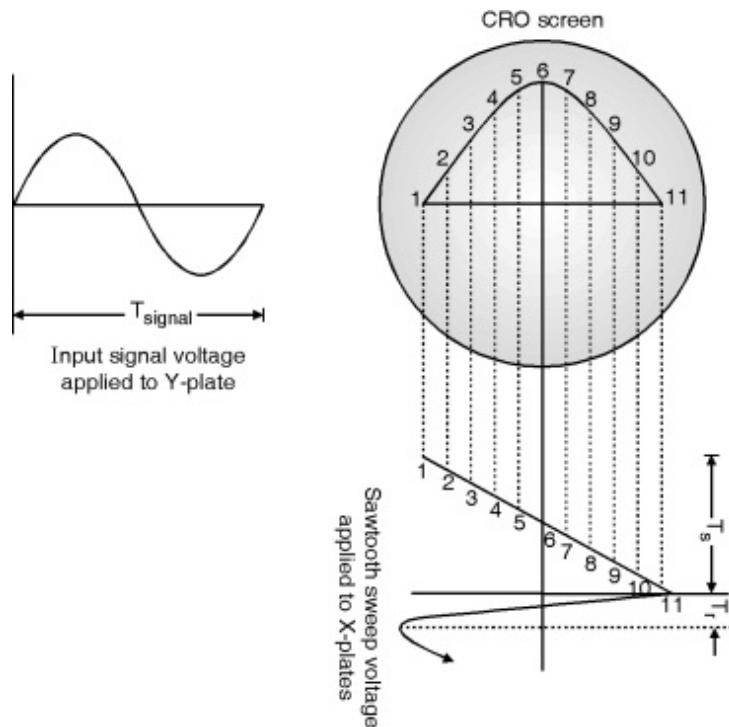
Fig. 7.14.5(b) : When $T_{signal} = 2 \, T_{sweep}$

Synchronization

- In Fig. 7.14.5(a) and Fig. 7.14.5(b) it must have been observed by the reader that horizontal sweep signal to x-plate deflection must be applied at the same time when input signal on vertical deflection (Y plate) in order to have stationary wave pattern.

- In this case sweep signal is said to be synchronized with input signal. If this type of synchronization is not achieved, wave pattern on CRO screen continuously drifts either on right or left of the screen.

Trigger circuit

- An electronic circuit is required to lock the frequency of time based circuit with the frequency of signal under

investigation which is applied to Y – plates is called triggered circuit.

- In trigger mechanism, a part of the output obtained from vertical amplifier is fed to trigger generator. When a predetermined level is reached, trigger generator will produce a pulse and this pulse which acts as a command to generate one sweep cycle of the time base.

Power Supply

Various sections of CRO require different magnitudes of electric potentials. Hence various power supply circuits are designed. Basically these circuits are divided into two main categories,

(a) Low voltage supply

- It provides electric supply to electronic circuits such as amplifiers trigger generator, time base generator etc.
- All these section require few tens of volts.

(b) High voltage power supply

- It provides voltages to electrodes in the CRT i.e. heater, accelerating and focussing mechanism.
- Magnitude of voltage supplied are around few kilo volts.

Syllabus Topic : Applications of CRO : Voltage (dc, ac), Frequency, Phase Measurement

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Q. Write any two applications of CRO. (Dec. 2013)

Ans. :

Applications of CRO

(a) Measurement of D.C. Voltage

(b) Measurement of A.C. Voltage and Frequency

(c) Measurement of Phase Difference

Q. Explain the measurement of frequency of AC signal using CRO.(May 2013, May 2014, May 2016)

Ans. :

Measurement of A.C. Voltage and Frequency

(1) The input coupling switch is kept in A.C. position. The function generator's mode is kept as sinusoidal. The output of the function generator is connected to CH 1 of CRO using BNC connectors.

(2) The trace on the screen is made to coincide with the central horizontal line of the graticule. The function generator is then turned ON. A signal of a particular frequency is generated using the frequency selector knobs of the generator. The amplitude is set using the amplitude knob.

(3) By adjusting the vertical amplifier sensitivity and time/div knob on the CRO the wave size on the graticule is adjusted to get a sufficiently large display without going beyond the limits of the screen.

(4) The vertical length of the wave from the negative maximum to the positive maximum is read on the graticule. It is multiplied by the value set on the volt/div knob. This will give us the peak to peak value (amplitude) of the A.C. signal. Then observe the length of one full cycle of the wave on the CRO in the horizontal direction. Multiplying this value with the value set on the time/div knob we get the time period of the A.C. signal. Reciprocal of this value will

give us the frequency of the A.C. signal (Refer Fig. 7.15.3).
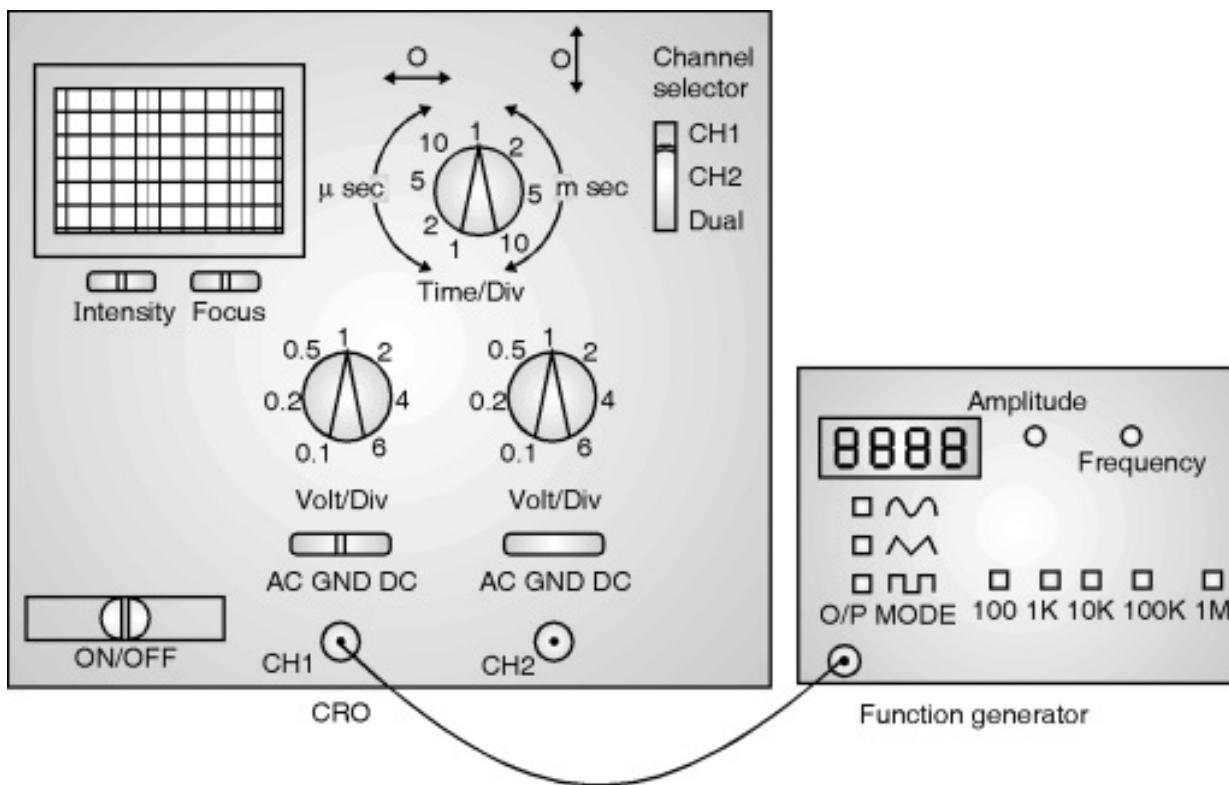
Fig. 7.15.3 : AC volts measurement

Vertical displacement $=$ 5.6 div

Volt/div knob is at 1 volt/div

$\therefore$ Amplitude of the applied signal $=$ 5.6 div $\times$ 1 volt / div

$=$ 5.6 Vp.p

horizontal displacement of one complete cycle $=$ 3.2 div.

$\therefore$ time period of the signal $=$ 3.2 div $\times$ 2 m sec/div

$=$ 6.4 $\times$ 10^{-3} sec

$\therefore$ Frequency of the signal $=$ 1/6.4 $\times$ 10^{-3} $=$ 156.25 Hz
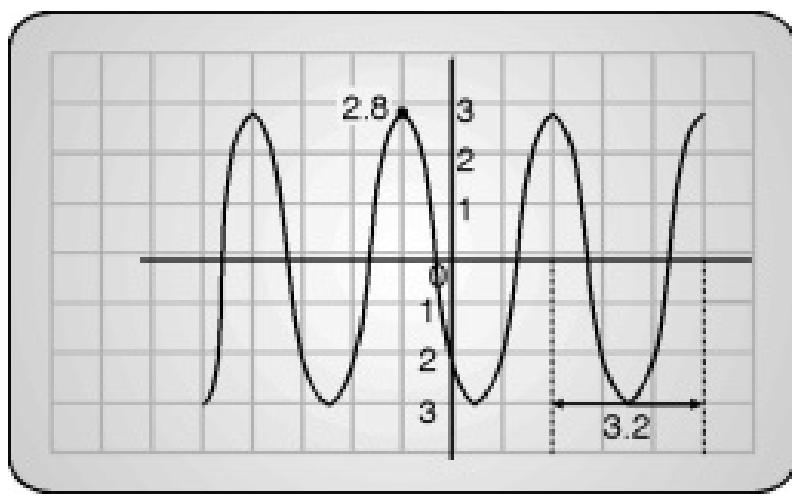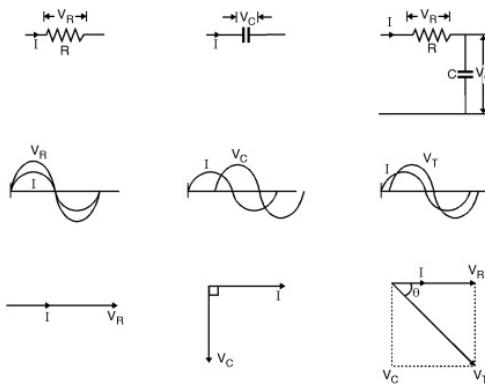
Fig. 7.15.4

Q.   How phase difference between two signals is measured using CRO ?(May 2015, Dec. 2015)

Ans. :

- The circuit connection for performing the phase shift experiment shown in Fig. 7.15.4. A resistor is connected in series with a capacitor and an A.C. voltage is applied to the RC network.

(a) Across resistor   (b) Across capacitor   (c) Across RC network

Fig. 7.15.5 : Phase relationships between voltage and current

- The voltage form the A.C. source is directly applied at CH1

and voltage across the capacitor is applied at CH2. The A.C. voltages have the same frequency but differ in their phases as the RC network introduces a finite phase difference between them.

- The resistance R may be made variable so that the phase difference between the voltages can be varied. Usually a decade resistor box is employed for the purpose.

(i) Direct method

(1) The phase shift circuit is connected into the circuit along with the CRO and signal generator as shown in the Fig. 7.15.6. The RC network introduces a phase difference between the signals fed to CH1 and CH2.

(2) The CRO is kept is duel mode and the function generator which is kept in sinusoidal mode is switched ON. The signal from the function generator is kept at a particular amplitude and frequency using the frequency selector knob.

(3) Two signals of the same frequency are displayed on the screen. It is seen that the phases of the signals are different.

(4) The vertical position settings of CH1 and CH2 are adjusted such that the two waves overlap on each other.
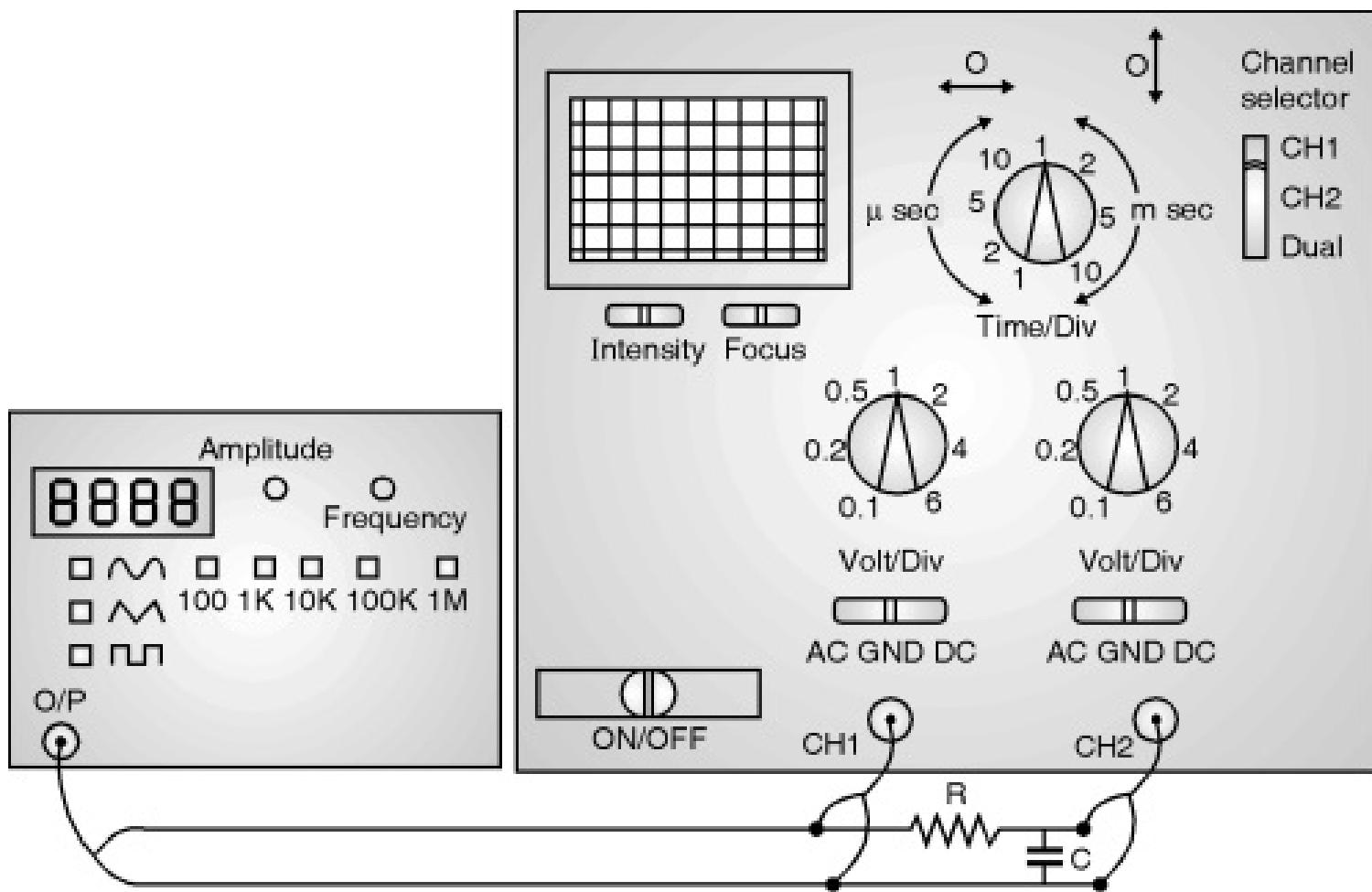
Fig. 7.15.6

(5) The sweep speed in initially adjusted such that the period T of the sine wave is measured (same as the procedure discussed earlier). Fig. 7.15.7(a).
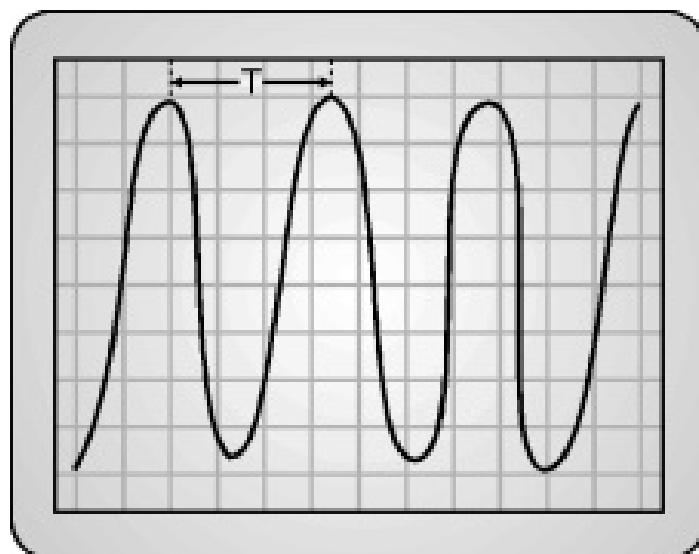
Fig. 7.15.7(a)

(6)   The sweep speed is increased and the delay time T_d between the two sine waves in accurately measured. Fig. 7.15.7(b).
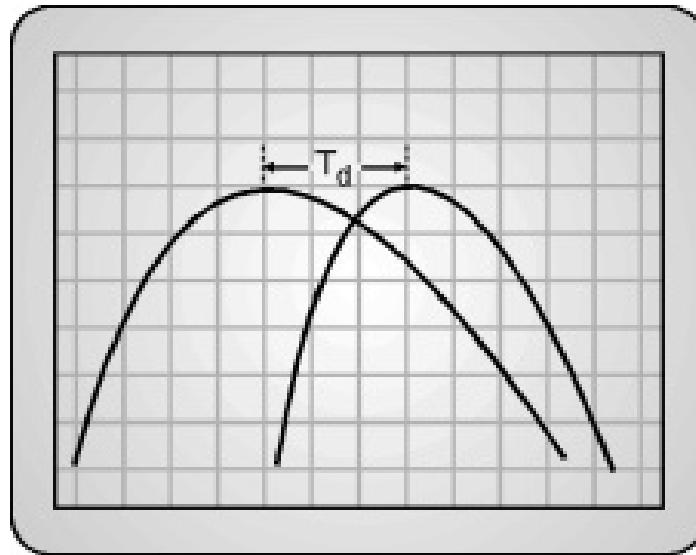
Fig. 7.15.7(b)

(i) $\theta_{exp} = \dfrac{T_d}{T} \times 360°$

(ii) $\theta_{the} = \tan^{-1}(2\pi fCR)$

(Where θ_{exp} is the measured phase difference and θ_{the} is the calculated phase difference)

(7)   The resistance value is changed by altering the setting of the dial of the decade box. The phase difference is again determined and measured.

(ii) XY method

(1)   This method makes use of lissajous pattern.

(2)   The time/div knob is set to X-Y EXT state. In this state, the signal given to CH1 acts as the sweep signal.

(3) The vertical mode switch is kept in X-Y position. An ellipse such as the one illustrated in Fig. 7.15.8 is obtained on the CRO screen.
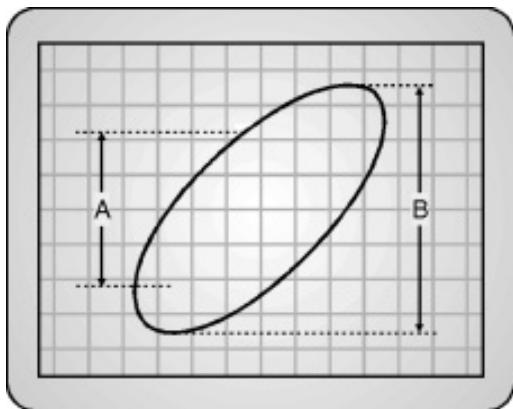
Fig. 7.15.8

(4) The AC – GND – D.C. switch is set to GND position.

(5) The vertical lengths. A and B of the ellipse are measured.

$$\theta = \sin^{-1}(A/B)$$

$$= \sin^{-1}(2.4/4)$$

$$= 36.86°.$$

(6) The value of the resistor R is varied by changing the dial setting on the decade box and measurements are taken one again.

Solved Problems

Ex. 7.16.1

In a CRT the distance from screen to the center of the coils is 0.2 m. The length of the magnetic field along the axis is 5 cm. Calculate the flux density B required to produce a deflection of 1 cm on the screen. If the anode voltage is 1000 volt.

Soln. :

Data : $D = 0.2$ cm

$l = 5 \times 10^{-2}$ cm

$Y = 1 \times 10^{-2}$ cm

$V_A = 1000$ V

Formula :

$$Y = D l B \sqrt{\frac{e}{2m} \cdot \frac{1}{V_A}}$$

$$\therefore B = \frac{Y}{Dl} \cdot \frac{1}{\sqrt{\frac{e}{2m} \cdot \frac{1}{V_A}}}$$

$$= \frac{1 \times 10^{-2}}{0.2 \times 5 \times 10^{-2}} \cdot \frac{1}{\sqrt{\frac{1.6 \times 10^{-19}}{2 \times 9.1 \times 10^{-31} \times 1000}}}$$

$$\therefore B = 1.066 \times 10^{-4} \text{ Wb/m}^2$$

...Ans.

Ex. 7.16.2

A CRT, employing magnetic deflection has a length of field equal to 3 cm along tube axis and flux density equal to $1 \times 10^{-4} \frac{Wb}{m^2}$. The distance form the centre of the field to the screen is 20 cm. The final anode voltage is 800 volts. Calculate the deflection of an electron in cm. Also find the displacement if a particle with charge twice that of an electron and mass $\sqrt{344}$ times as large. Given e/m = 1.76×10^{11} C/kg

Soln. :

Data : $l = 3$ cm

$B = 1 \times 10^{-4} \text{ Wb/m}^2$

$D = 20$ cm

$V_A = 800$ Volts

$e/m = 1.76 \times 10^{11}$ C/kg

Formula :

$$Y = D l B \sqrt{\frac{e}{2m} \cdot \frac{1}{V_A}}$$

$$= 20 \times 10^{-2} \times 3 \times 10^{-2} \times 1 \times 10^{-4} \times \sqrt{\frac{1.76 \times 10^{11} \times 1}{2 \times 800}}$$

$$\therefore \qquad \text{Deflection} = 6.3 \times 10^{-3} \text{ m} \qquad \text{...Ans.}$$

(ii) For the charge particle with charge twice that of electron and mass 7344 times.

$\therefore$ Using the same formula

$Y = D/E \sqrt{\frac{2e}{2 \times 7344 \, m} \cdot \frac{1}{V_A}}$

$$= 20 \times 10^{-2} \times 3 \times 10^{-2} \times 1 \times 10^{-4} \times \sqrt{\frac{2 \times 1.6 \times 10^{-19}}{2 \times 7344 \times 9.1 \times 10^{-31} \times 800}}$$

$$= 1.038 \times 10^{-4} \text{ m}$$

$\therefore$ Displacement $\quad = 0.01$ cm $\qquad$ ...Ans.

Ex. 7.16.3

An electrically deflected CRT has a anode voltage 2000 V and parallel deflecting plates 1.5 cm long and 5 mm apart. If the screen is 50 cm from the center of deflecting plates find (a) Beam speed (b) The deflection sensitivity of the tube and (c) Deflection factor of tube.

Soln. :

Data : $\qquad\qquad V_A = 2000$ V

$l = 1.5$ cm $= 1.5 \times 10^{-2}$ m

$d = 5$mm $= 5 \times 10^{-3}$ m

$D = 50$ cm $= 50 \times 10^{-2}$ m

(i) Velocity of beam

Using concept $KE = \frac{1}{2} mv^2 = eV_A$

$$\therefore \quad v = \sqrt{\frac{2eV_A}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2000}{9.1 \times 10^{-31}}}$$

Velocity $= 26.5 \times 10^6$ m/sec $\qquad$ ...Ans.

(ii) Deflection sensitivity

$$S = \frac{y}{D} = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{1}{V_A}$$

$$= \frac{0.5 \times 1.5 \times 10^{-2}}{2 \times 5 \times 10^{-3} \times 2000}$$

$$= 3.75 \times 10^{-4} \text{ m/volt}$$

$\therefore$ Deflection sensitivity $= 0.375$ mm/volt ...Ans.

(iii) Deflection factor

$$D = \frac{1}{S} = \frac{1}{0.375} = 2.67 \text{ V/mm} \qquad \text{...Ans.}$$

Ex. 7.16.4

A CRT has anode voltage 1000V and parallel deflecting plates 2cm long and 5 mm apart. the screen is 30cm from the center of the plates. Find the input voltage required to deflect the beam 1.5 cm. The input voltage is applied to the deflecting plates through amplifiers having an overall gain of 100.

Soln. :

Data : $\qquad V_A = 1000$ V

$l = 2$ cm $= 2 \times 10^{-2}$ m

$d = 5$ mm $= 5 \times 10^{-3}$ m

$D = 30$ cm $= 0.3$ m

$Y = 1.5$ cm $= 1.5 \times 10^{-2}$ m

Formula : $\qquad Y = \frac{D}{2} \cdot \frac{l}{d} \cdot \frac{V}{V_A}$

$$\therefore \quad V = \frac{2 d V_A}{D l} \cdot Y$$

$$= \frac{2 \times 5 \times 10^{-3} \times 1000 \times 1.5 \times 10^{-2}}{0.3 \times 2 \times 10^{-2}} = 25 \text{ Volt}$$

Input voltage required for a deflection of 1.5 cm

$$= \frac{V}{\text{gain}} = \frac{25}{100}$$

$$= 0.25 \text{ Volt} \qquad \qquad \text{...Ans.}$$

Ex. 7.16.5

A CRT has anode voltage of 1.6 kV and parallel deflection plates are 1.8 cm long and 4.2 mm apart. The screen is at 2.8 cm from the centre of deflecting plates. Find the input voltage required to deflect the beam by 1.5 cm. The input voltage is applied to the deflection plates through the amplifiers having overall gain 60.

Soln. :

$$y = \frac{D}{2} \cdot \frac{l}{d} \frac{V}{V_A}$$

$$V = \frac{2yd\,V_A}{D\cdot L} = \frac{1.5 \times 2 \times 4.2 \times 1.6 \times 10^3 \times 10^{-3}}{2.8 \times 1.8}$$

$$= 4V$$

As

$$\frac{V}{V_{app}} = \text{gain}$$

$$V_{app} = \frac{V}{\text{gain}} = \frac{4}{60} = 0.0667 \text{ V} \qquad \text{...Ans.}$$

Ex. 7.16.9 (Dec. 2014, 3 Marks)

In phase measurement by Lissajous pattern ellipse is obtained with major axis of 2 cm and minor axis of 0.8 cm. Calculate phase change.

Soln. :

$$\text{Phase angle} = \sin^{-1}\left(\frac{\text{Minor axis}}{\text{Major axis}}\right) = \sin^{-1}\left(\frac{y_1}{y_2}\right)$$

$$= \sin^{-1}\left(\frac{0.8}{2}\right)$$

$$= 23.57° \qquad \qquad \text{...Ans.}$$

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Syllabus Topic : Introduction to Nano-science and Nanotechnology

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Q.   What are nano materials ? (Dec. 2016)

Ans. :

Nano Materials

    If we take a material in which the atoms do not move away from each other and with size in the range of 1 to 100 nano meters, these materials are called nano materials.

Syllabus Topic : Surface to Volume Ratio, Two Main Approaches in Nanotechnology-Bottom Up Technique and Top Down Technique

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Q.   Explain the top down approach and bottom up approach to prepare nanomaterials.

       (May 2015, Dec. 2015)

Ans. :

(i)   Bottom up approach

    In this, nano materials are made by building atom by atom or molecule by molecule.

(ii)   Top down approach

In this a bulk material is broken or reduce in size or pattern. The techniques developed under this tile are modified or improved one what we have in use to fabricate micro-processors, Micro-Electro-Mechanical Systems (MEMS) etc.

Q.   With neat diagram, explain construction and working of transmission electron microscope.

Ans. :

Electron Microscope

- Electron microscope makes use of electron – a negatively charged particle, in terms of its wave like properties (quantum mechanical aspects). Electrons possess two main advantages for use in microscopy. Electron Microscope

- According to quantum physics, the wavelength associated with electron varies inversely as its energy.

$$\lambda = \frac{h}{\sqrt{2\,mE}}$$

- i.e. with increase in the energy, wavelength can be reduced. This brings in higher resolving power and hence better magnification, (nearly 1000 times that of optical microscope).

- Electron can be focussed by means of magnetic lenses unlike electromagnetic radiations.

- The first electron microscope which was a TEM (Transmission Electron Microscope) was schematically as shown below. For a student, comparison between electron microscope and optical microscope is advisable.

- Fig. 8.3.1 shows schematic diagram of TEM.
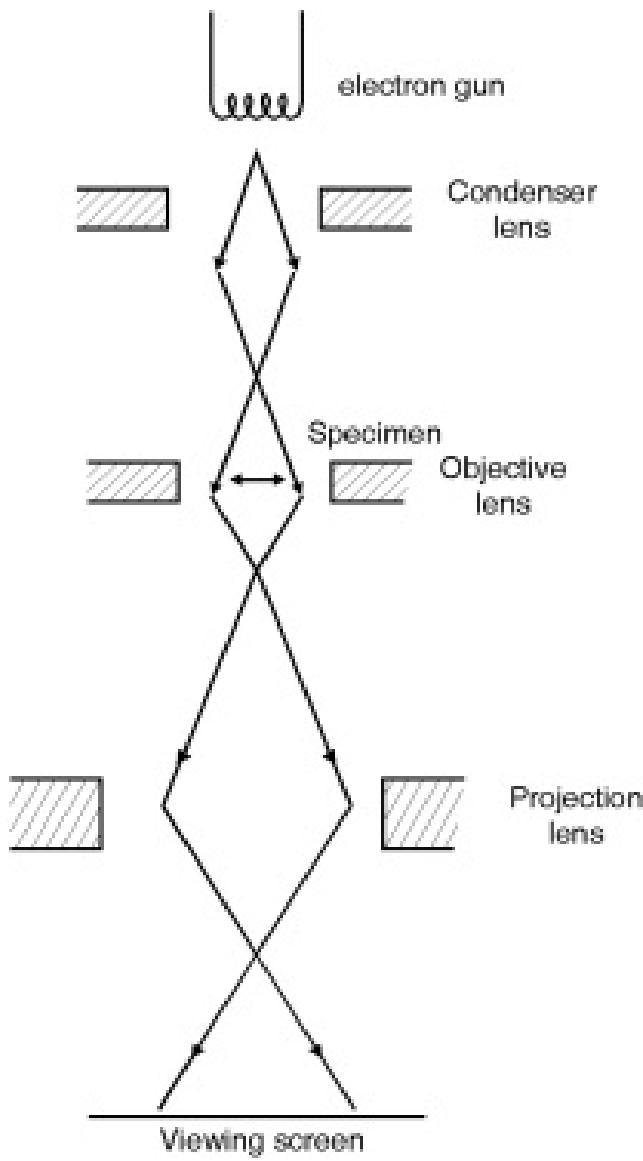
Fig. 8.3.1

- The source of electrons will provide a flow of electrons whose energy and hence the wavelength can be varied.

- The beam path, the placement of the lenses, the specimen, the aperture etc. follow the plan of the light microscope. The entire arrangement is placed in high vacuum to avoid extraneous scattering and absorption of the electrons by air.

- The lenses are magnetic lenses and the magnified image is

projected on a fluorescent screen. The extra aperture called the objective aperture is placed in electron microscope which not there in optical microscope.

- In an optical microscope light is transmitted or absorbed by the specimen. This creates a contrast between different parts of the image. Here incident beam is passed through the specimen with very little absorption.

Q.   Draw the schematic diagram of SEM and explain its construction and working. (May 2013, Dec. 2013, May 2014, Dec. 2014)

Ans. :

SEM (Scanning Electron Microscope)

- Unlike TEM mentioned above, SEM (Scanning Electron Microscope) is used to obtain images of surface of thickness. Also a thin specimen can be studied.
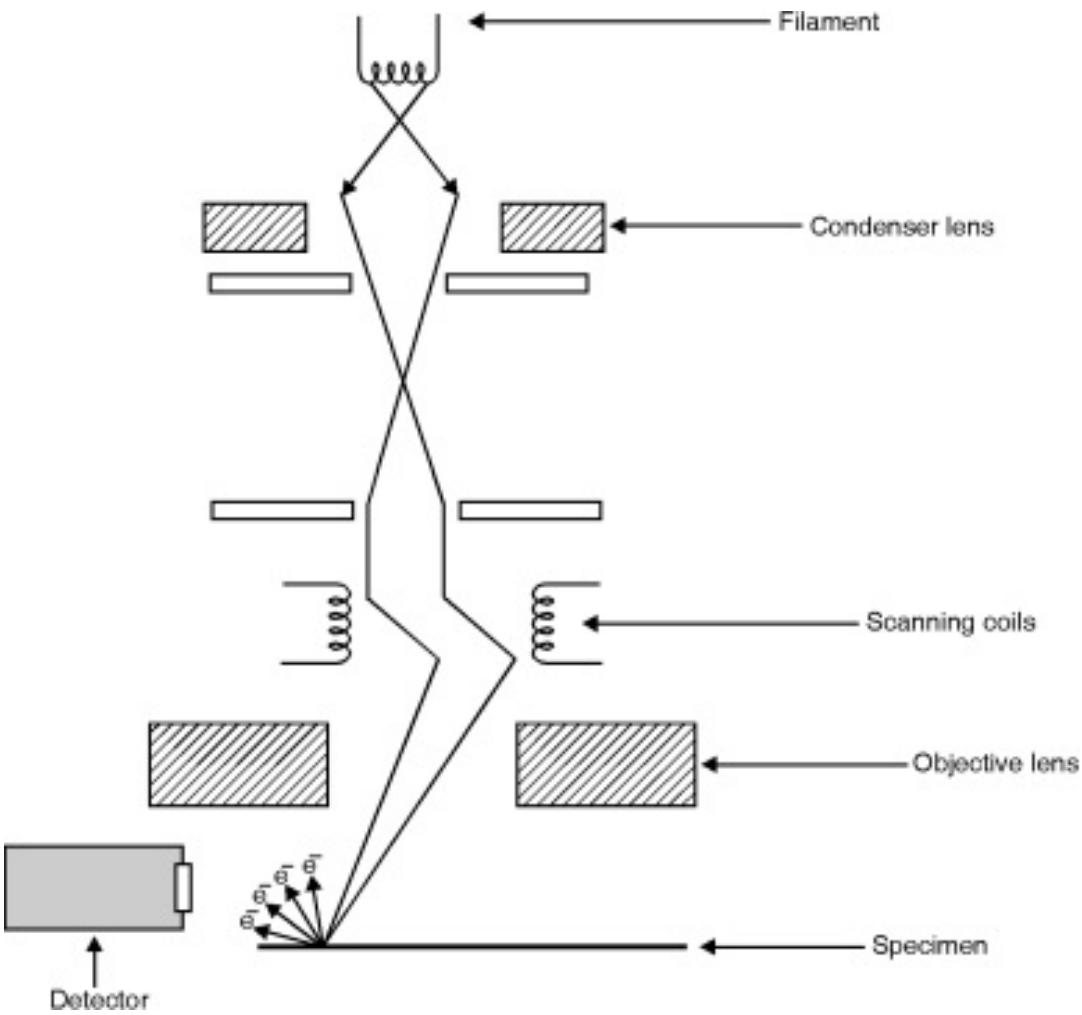
Fig. 8.3.2 : Schematic diagram of SEM

- Construction of TEM includes an arrangement that makes it possible for an electron beam to scan the specimen similar to that we have in TV picture tube.

- Construction of SEM is shown in Fig. 8.3.2. Here electron beam is obtained from electron gun and it is made to pass through condenser lens. Next stage is of scanning coil which is used to focus the electron beam on a small spot on specimen surface and also to scan the surface like electron beam scans in TV picture tube.

- Electrons, on reaching the surface of specimen, will be scattered. The scattered beam will form the image signals.

- Detector is kept on the same side where other components of SEM are kept.

- Image formation in SEM is due to two main combining aspects.

  (a) Scattering of electron beam is because of atoms on the surface of the specimen and these atoms have different scattering power.

  (b) Topographical variations of the surface.

  Actually, the aspects mentioned above are also responsible for the contrast which is essential for image formation.

- During the scanning of atoms by electron beam, the scattered electrons intensities are measured by detector and then displayed on the screen. If the scattering is high at particular point during the scanning, the corresponding point on the viewing screen will be bright and for low scattering, the corresponding point on the screen will be dark. This develops required contrast for a clear image of the specimen.

- Specimen as small as 50 A° size may be clearly resolved by SEM.

Q. With neat diagram, explain construction and working of atomic force microscope.(May 2015, May 2016, Dec. 2016)

Ans. :

Atomic Force Microscope (AFM)

- Atomic Force Microscope is high resolution type of Scanning probe microscope with resolution of 1 A.U. Because of this it is one of the foremost tool in the field of

nano science.

- Atomic Force Microscope (AFM) is a modified TEM to overcome limitations of TEM. The needle mentioned in TEM which works as the probe is kept in touch with sample using a micro scale cantilever.

- When the tip is brought in touch with the sample surface, forces between the tip and the sample lead to the deflection of the cantilever.

- The force present in the tip is kept constant and the scanning is done. As the scanning continues, the tip will have vertical movements depending upon topography of the sample.

- The tip has a mirror on top of it, a laser beam is used to have the record of vertical movement of the needle. Interferometer is also used for best of accuracy.

- The information is later converted to a visible one.

- It overcomes the difficulty of TEM i.e. the problem associated with non conducting material as AFM does not generate any current.

- Depending on the situation forces that are measured in AFM include mechanical contact force, Vander Walls force, capillary forces, electrostatic and magnetic forces.

Limitations

Since the tip of the needle is in direct touch with the sample surface, it is likely to distort the biological samples.

Q.   Explain the difference between SEM and AFM.

Ans. :

Differences between SEM and AFM

Sr. No.	SEM	AFM
1.	Provides a two dimensional projection.	Provides 3D profile of the specimen.
2.	Requires a special treatment to the sample which has the potential to either damage the sample or change some of the property.	Samples viewed by AFM does not require special treatments like coating etc.
3.	Requires a vacuum environment for its operation.	Can work very well in air.
4.	Living cells can not be studied due to requirement of vacuum.	Living cells can be studied.
5.	The quality of the resolution is better.	The quality of the resolution is restricted by radius of curvature of the tip of the needle.

Q.   What are different techniques to synthesis nanomaterial? (May 2013)

Ans. :

Different techniques of Synthesis Nanomaterial

Various techniques are adopted for the synthesis of nano materials that too in various forms like nano particles, nano powder, nano crystals, nano films, nano wires, nano tubes, nano dots. These methods includes:

1.   Ball Milling

2.   Sputtering

3.   Vapor deposition

4.   Sol gel technique.

5.   Electro deposition.

6.   Mechanical crushing or Ball milling

7.   LASER synthesis

8.   Inert gas condensation

Q.   Explain Ball Milling technique in detail for synthesis of nano particles. (May 2013, Dec. 2016)

OR

Q.   Explain the Physical Methods for synthesis of Nanoparticles. (May 2014)

Ans.:

Ball Milling

- It is a process where a powder mixture placed in the ball mill is subjected to high energy collision from the balls.

- Planetary ball mill is most frequently used system for mechanical alloying since only a very small amount of powder is required. In simple language, a ball mill consists of a hollow cylindrical shell rotating about its axis.

- The axis of the shell may be either horizontally or at a small angle to the horizontal. It is partially filled with balls which makes grinding media and made up of steel, stainless steel or ceramic.

- The inner surface of the shell is made up of an abrasion resistant material. When continuously operated, the shell rotates and lifts the balls up and drops them from near the top of the shell which causes the grinding of the particles inside.

Q.   Explain Sputtering technique in detail for synthesis of nano

particles. (Dec. 2016)

OR

Q. Explain the Physical Methods for synthesis of Nanoparticles. (May 2014)

Ans. :

Sputtering

- Sputtering is a process whereby particles are ejected from a solid target material due to bombarding of target by energetic particles. It is necessary to have kinetic energy of incoming particles much greater than conventional thermal energies.

- In this technology, the substrate is placed in a vacuum chamber with source material, named target, and an inert gas(such as Argon) is introduced at low pressure. A gas plasma is struck using an RF power source, causing the gas to become ionized.

- The ions are accelerated towards the surface of the target, causing atoms of the source material to break from the target in vapour form and condense on all surfaces.

Syllabus Topic : Vapour Deposition

Q. Explain vapour deposition technique in detail.

Ans. :

Vapour Deposition

- This method is used to prepare nano powder.

- In this technique initially the material is heated to form a gas and is allowed to deposit on a solid surface under

vacuum condition which forms nano powders on the surface of the solid.

Syllabus Topic : Sol Gel

Q. Explain Sol Gel technique in detail.

Ans. :

Sol Gel

- Nano particles and nano powder is obtained using this technique.

- In general, sol - gel technique is based on the hydrolysis of liquid precursors and formation of colloidal solutions.

- Out of few more processes, hydro - dynamic cavitation is often used, in which nanoparticles can be generated through creation and release of gas bubbles inside the sol - get solution.

- Here, the sol - get solution is taken in a drying chamber and thoroughly mixed by applying enormous pressure, high temperature and further exposing it to cavitational disturbances.

- This process creates hydrodynamic bubbles in the sol - gel. These bubbles will undergo nucleation, growth and then it quenches to form nano particles.

Syllabus Topic : Properties and Applications of Nanomaterials

Q. Enlist various properties of nanomaterials.

Ans. :

Properties of Nano Particles

Nano materials are materials possessing grain size of the order of a billionth of the meter. Few of the properties are

1. Because of very small size, nano particles cannot be further divided into smaller ones and it cannot have any dislocations. Therefore we can describe them with (a) hard and (b) wear resistant

2. They are ductile at high temperature.

3. Exhibit very low wear and tare

4. Active for chemical reactions.

5. Less corrosion.

Though nano particles are very small, they are the important materials to built the future world. They have applications almost in all engineering field.

Q.   Explain applications of nanomaterials in various fields.

Ans. :

Applications of Nanomaterials in Various Fields

Applications in Mechanical Engineering

(1) Since they are stronger, lighter etc., they are used to make hard metals.

(2) Smart magnetic fluids are used in vacuum seals, magnetic separators etc.

(3) They are also used in Giant Magneto Resistance (GMR) spin valves.

(4) Nano MEMS (Micro Electro Mechanical Systems) are used in optimal switches, pressure sensors, mass sensors.

(1) Orderly assembled nano materials are used as quantum electronic devices and used in photonic crystals.

(2) Some of the nano materials are used as sensing elements.

Especially the molecular nano materials are used to design robots, assembler etc.

(3) They are used in energy storage device such as hydrogen storage device in ionic batteries.

(4) In magnetic recording devices.

(1) To make CDs, and semiconductor LASER

(2) To make smaller chips few information storage.

(3) In mobile phones, Lap - tops etc.

(4) Nano dimensional photonic crystals and quantum electronic crystals and quantum electronic devices play a vital role in recently developed computers.

(1) Consolidated state nano particles are used as catalysts, electrodes in solar and fuel cells.

(2) Bio- sensitive nano particles are used in the production of DNA chips, bio-sensors etc.

(3) Nano - structured ceramic materials are used in synthetic bones.

(4) Few nano materials are used in absorbents, self cleaning glass fuel additives, drugs, ferro fluids etc.

(5) Nano metallic colloids are used as film precursors.

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